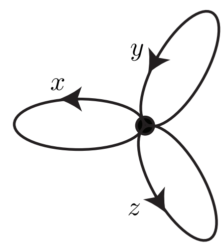
RICE UNIVERSITY TOPOLOGY QUALIFYING EXAM - MAY 2012

This is a 3-hour closed book, closed notes exam. Please show all of your work and justify as many statements as time allows.

1. Let $W = S^1 \vee S^1 \vee S^1$ as shown below. Let x, y, z be the three loops indicated going around the first, second, third circles respectively. Let $X = W \cup_{f_1} e_1^2 \cup_{f_2} e_2^2$ be the space obtained from W by adjoining one 2-cell via the map f_1 which forms the loop $xyx^{-1}z^3y^{-1}$; and another 2-cell via the map f_2 which forms the loop z^9.



a) Give a cellular chain complex for X (including the boundary maps).
b) Compute $H_p(X; \mathbb{Z}_9)$ for all p.
c) Compute $H^p(X; \mathbb{Q})$ for all p without using a universal coefficient theorem.

2. Let A and B be non-empty subsets of the sphere S^n, $n \geq 2$. Prove: If A and B are closed, disjoint , and neither separates S^n, then $A \cup B$ does not separate S^n. (recall that C separates S^n means $S^n - C$ is disconnected).

3. Prove that any continuous map $f : \mathbb{C}\mathrm{P}(4) \to S^3 \times S^5$ induces the zero map on $H_8(\ ;\mathbb{Z})$. Justify statements.

4. a) Suppose that G is a finitely-generated group. Prove that there exists a connected 1-dimensional CW-complex on which G acts freely and properly discontinuously such that the quotient space is compact.
b) Construct a connected 1-dimensional CW-complex on which the symmetric group S_3 acts freely and properly discontinuously and describe the quotient space.

5. Let X and Y be compact, connected, oriented n-dimensional manifolds without boundary and let $f : X \to Y$ be a continuous map with degree 1. Prove that $f_* : \pi_1(X) \to \pi_1(Y)$ is surjective. (Hint: suppose the image of f_* is a proper **subgroup** of $\pi_1(Y)$.) Your proof should include as many details as time allows.