The General Qualifying Exam in Analysis

The main focus of the general qualifying examination in analysis is beginning graduate real and complex analysis. However, it is essential for students to have also a thorough and working knowledge of what might be called basic undergraduate analysis. The (inexhaustive) list of topics below is intended to provide students with both a guide for preparing for the general qualifying examination in analysis and a sampling of important undergraduate material that graduate students should know already and which ought to be reviewed or studied as necessary.

It should be emphasized that it is not sufficient for students to know simply the statements of the major definitions and results and something about their proofs. A practical working knowledge is required; students should understand why important definitions and theorems are important, how they fit together (and not just within a particular area), how to recognize when various concepts or results are relevant (even when this is not explicitly given), why the hypotheses of specific theorems are needed, etc. In preparing for future study of mathematics students are urged to work many and varied problems and to study examples.

Basic Undergraduate Analysis

Basic reference: Rudin, *Principles of Mathematical Analysis.*

Important topics: Sequences and series, convergence, compactness, continuity, uniform continuity, Riemann integrals (including iterated and multiple integrals), uniform convergence (especially in connection with continuity, integration, and differentiation), power series, basic facts related to linear algebra (especially in connection with eigenvalues, eigenvectors, and the diagonalizability of real symmetric matrices), Green's theorem, the inverse and implicit function theorems, the existence and uniqueness theorem for ordinary differential equations, and the Arzela-Ascoli theorem.

Real Analysis

Basic reference: Royden, *Real Analysis.*

Important topics: Lebesgue measure, Lebesgue integrals, various notions of convergence of functions (e.g., pointwise a.e., in measure, in norm) and their relationships with each other, Egoroff's theorem, convergence theorems for integrals, the Minkowski and Hölder inequalities, L^p spaces, the completeness of the L^p spaces, the various approximation theorems (e.g., approximation of measurable functions by simple functions, or by continuous functions, the denseness of continuous functions inside L^p spaces when $p < \infty$, etc.), product spaces and Fubini's theorem.

Complex Analysis

Basic reference: Ahlfors, *Complex Analysis.*

Important topics: Holomorphic functions, Cauchy's theorem (in its various forms), the Cauchy integral formulae, Liouville's theorem, Morera's theorem, the maximum principle, Schwarz's lemma, local properties of holomorphic functions, the reflection principle, the residue calculus, the argument principle and its applications, Taylor series and Laurent series, uniform convergence and holomorphicity, Hurwitz's theorem, linear fractional transformations, and the Riemann mapping theorem.