THE FIRST MIDTERM
REPRESENTATION THEORY 374.

3 problems for a C; 7 problems for a B; 9 problems for an A.
Each problem with asterisks counts as two.
In all problems, please give detailed solutions.
Please clearly formulate all theorems that you are using.
Consultation with other people is not allowed.
You may use any materials.
No credit for wrong answers.
Please write the Honour Pledge in full.
This midterm is due on Tuesday 8 MARCH in class.

I. The group $A_4$.
Consider the group $A_4$ of even permutations of four symbols.
1. Find all the conjugacy classes in $A_4$.
2. Classify all the irreducible complex representations of $A_4$.
4. ** For $n \geq 5$, find all one-dimensional representations of the group $A_n$.
5. ** Let $T_1, T_2, \ldots, T_q$ be irreducible representations of $A_4$. Consider all of
   the products $T_i T_j$, and decompose them into irreducibles.

II. Motions of Polyhedra.
Let $M$ be a polyhedron in $\mathbb{R}^3$. Let $O(M)$ be the group of isometries
of $\mathbb{R}^3$ preserving $M$, and let $SO(M)$ be the subgroup of isometries with
determinant one (recall that all isometries of $\mathbb{R}^3$ are linear maps).
Let $T$ be a regular tetrahedron, $C$ a cube, and $O$ a regular octahedron.
6. Find $O(T)$ and $SO(T)$.
7. Find $O(C)$, $O(O)$, $SO(C)$, and $SO(O)$.
8. For $M = T$, $C$, $O$, show that the identity representations of $O(M)$ and
   $SO(M)$ in $\mathbb{R}^3$ are irreducible.

III. The group $H$.
Let $H$ be the group with eight elements: $1, i, j, \varepsilon, \varepsilon i, \varepsilon j, ij, ji$, with mul-
tiplication determined by the rules $i^2 = j^2 = \varepsilon^2 = 1, \varepsilon i = i \varepsilon, \varepsilon j = j \varepsilon,$
$ij = \varepsilon ji$. 

9. Find the commutator subgroup of $H$.

10. Find the number of irreducible complex representations of $H$.

11. ** Draw the character table of $H$.

IV. Real representations of cyclic groups.

12. ** Find all irreducible real representations of the cyclic group $\mathbb{Z}/n\mathbb{Z}$.