MATH 366: Assignment 4
Due Friday, February 3, 2012

Incidence geometry

1. Classify up to isomorphism the incidence geometries with exactly four points (i.e. give a list of examples of incidence geometries with exactly four points, show that they aren’t isomorphic to each other, and show that every incidence geometry with exactly four points is isomorphic to one of them).

2. Do exercise 14 and major exercises 3, 5, \(4\) from chapter 2 in the textbook.

   [Remark: It is conjectured that there exists a finite projective plane of order \(n\) only if \(2^n\) is a power of a prime, but even the case \(n = 12\) remains an open problem. Part of the proof that there are no finite projective planes of order 10 required a vast computer search.]

Extra credit

3. In the affine plane \(\mathbb{Q}^2\) of points with rational coordinates and the lines whose equations have rational coefficients, we can give the undefined notions of congruence of segments and angles their usual meaning.\(^3\)

   (a) Show that in this geometry, equilateral triangles do not exist (i.e. show that there is no equilateral triangle in the real Euclidean plane whose vertices all have rational coordinates).

   (b) Show that in this geometry, regular pentagons do not exist.

   (c) Which regular polygons exist in this geometry?

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\(^1\)The figure in the book for Major Exercise 8 is perhaps somewhat unclear as to what the lines are. See the color version on the course web page.

\(^2\)The converse is known: there exists a finite projective plane of order \(n\) if \(n\) is a power of a prime.

\(^3\)See page 139 for details on how to do so algebraically.
This figure shows the finite projective plane of order 3, with 13 points (the two colored points of each color should be identified with one another). Each black arc represents a single line, and each pair of same-colored arcs represents a single line.