B 14.1 By direct substitution, one checks that
\[ e^t \cos t, \; e^t \sin t, \; e^t \cos t + e^t \sin t \]
satisfy \[ y'' - 2y' + 2y = 0 \]

B 14.1 By direct substitution, one can check that
\[ y_1(t) = \cos 4t \quad \text{and} \quad y_2(t) = \sin 4t \]
are solutions of \[ y'' + 16y = 0 \]. Moreover, these functions are linearly independent (neither is a constant multiple of the other), one can also compute the Wronskian for this:
\[
W(\cos 4t, \sin 4t) = \begin{vmatrix} \cos 4t & \sin 4t \\ -4\sin 4t & 4\cos 4t \end{vmatrix} \\
= 4 \cos^2 4t + 4 \sin^2 4t = 4 \neq 0.
\]
The general solution is given by
\[ y(t) = c_1 \cos 4t + c_2 \sin 4t \]
We find \( c_1, c_2 \) from \( y(0) = 2, \; y'(0) = -1 \):
\[
y(0) = 2 \quad \Rightarrow \quad c_1 = 2
\]
\[
y'(0) = -1 \quad \Rightarrow \quad 4c_2 = -1 \Rightarrow c_2 = -\frac{1}{4}
\]
Hence \( y(t) = 2 \cos 4t - \frac{1}{4} \sin 4t. \)
$9.4.3. \ y'' + 4y' + 5y = 0$

Characteristic equation: $\lambda^2 + 4\lambda + 5 = 0$

$$\lambda = \frac{-4 \pm \sqrt{16 - 20}}{2} = \frac{-4 \pm 2i}{2} \rightarrow \lambda_1 = -2 + i \quad \lambda_2 = -2 - i$$

Fundamental set of solutions: $e^{-2t} \cos t, e^{-2t} \sin t$

General solution: $c_1 e^{-2t} \cos t + c_2 e^{-2t} \sin t$

$13.4.3. \ y'' - 4y' + 4y = 0$

$\lambda^2 - 4\lambda + 4 = 0$ has repeated root $\lambda = 2$

Fundamental set of solutions: $e^{2t}, t \cdot e^{2t}$

General solution: $c_1 e^{2t} + c_2 t \cdot e^{2t}$

$22.4.3. \ y'' + 10y' + 25y = 0 \quad y(0) = 2, \ y'(0) = -1$

$\lambda^2 + 10\lambda + 25 = 0$ has repeated root $\lambda = -5$

General solution $y(t) = c_1 e^{-5t} + c_2 t e^{-5t}$

From $y(0) = 2 \Rightarrow c_1 = 2$

$y'(0) = -1 \Rightarrow -5c_1 + c_2 = -1 \Rightarrow c_2 = 9$

Hence $y(t) = 2e^{-5t} + 9t \cdot e^{-5t}$
B 13/4.1. Similarly to the previous problem, by direct substitution, one shows that \( y_1(t) = e^{-4t} \), \( y_2(t) = te^{-4t} \) are solutions of \( y'' + 8y' + 16y = 0 \). These solutions are linearly independent, since the Wronskian is nonzero:

\[
W(y_1(t), y_2(t)) = \begin{vmatrix} e^{-4t} & te^{-4t} \\ -4e^{-4t} & e^{-4t} - 4te^{-4t} \end{vmatrix} = e^{-8t} - 4te^{-8t} + 4te^{-8t} = e^{-8t} \\
\]

Hence, the general solution is \( y(t) = c_1 e^{-4t} + c_2 te^{-4t} \).

Since \( y(0) = 2 \), \( y'(0) = -1 \), one can find \( c_1 = 2 \), \( c_2 = 7 \), and

\( y(t) = 2e^{-4t} + 7te^{-4t} \).

B 4/4.3. \( y'' + 5y' + 6y = 0 \)

Characteristic equation: \( \lambda^2 + 5\lambda + 6 = 0 \)

\[
\lambda = \frac{-5 \pm \sqrt{25-24}}{2} = \frac{-5 \pm 1}{2} \Rightarrow \lambda_1 = -3, \lambda_2 = -2
\]

Fundamental set of solutions: \( e^{-3t} \), \( e^{-2t} \)

General solution: \( c_1 e^{-3t} + c_2 e^{-2t} \).
28/4.3 \[ y'' - 4y' + 13y = 0 \quad y(0) = 4, \quad y'(0) = 0 \]

\[ \lambda^2 - 4\lambda + 13 = 0 \] has complex roots \( \lambda = 2 \pm 3i \).

The general solution: \[ y(t) = c_1 e^{2t} \cos 3t + c_2 e^{2t} \sin 3t \]

From \( y(0) = 4 \) \( \Rightarrow c_1 = 4 \)

From \( y'(0) = 0 \) \( \Rightarrow 2c_1 + 3c_2 = 0 \) \( \Rightarrow c_2 = -\frac{8}{3} \)

Therefore \( y(t) = 4 e^{2t} \cos 3t - \frac{8}{3} e^{2t} \sin 3t \).