HW #8 - Math 211

B 11/7.1  a = -3 , A = \begin{bmatrix} -1 & 2 & 0 \\ 3 & 0 & -1 \\ 2 & 2 & 1 \end{bmatrix}, \bar{x} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}; \bar{y} = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}

Need to verify: 1) \( A(a \bar{x}) = aA\bar{x} \)

\[ A(a\bar{x}) = \begin{bmatrix} -1 & 2 & 0 \\ 3 & 0 & -1 \\ 2 & 2 & 1 \end{bmatrix} (\begin{bmatrix} -3 \\ -2 \\ 0 \end{bmatrix}) = \begin{bmatrix} -1 & 2 & 0 \\ 3 & 0 & -1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 6 \\ 0 \end{bmatrix} = \begin{bmatrix} 15 \\ -9 \\ 6 \end{bmatrix} \]

\[ aA\bar{x} = (-3) \begin{bmatrix} -1 & 2 & 0 \\ 3 & 0 & -1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} = (-3) \begin{bmatrix} -5 \\ 6 \\ -2 \end{bmatrix} = \begin{bmatrix} 15 \\ -9 \\ 6 \end{bmatrix} \]

2) \( A(\bar{x} + \bar{y}) = A\bar{x} + A\bar{y} \)

\[ A(\bar{x} + \bar{y}) = \begin{bmatrix} -1 & 2 & 0 \\ 3 & 0 & -1 \\ 2 & 2 & 1 \end{bmatrix} (\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}) = \begin{bmatrix} -1 & 2 & 0 \\ 3 & 0 & -1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix} \]

\[ A\bar{x} + A\bar{y} = \begin{bmatrix} -1 & 2 & 0 \\ 3 & 0 & -1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 0 \\ 3 & 0 & -1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 15 \\ -9 \\ 6 \end{bmatrix} + \begin{bmatrix} -7 \\ -4 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 \\ -13 \\ 6 \end{bmatrix} \]

B 35/7.1. \begin{bmatrix} -6 & -1 & 7 \\ -1 & 8 & -9 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ -17 \end{bmatrix} = \begin{bmatrix} 20 \\ -17 \end{bmatrix} \hspace{1cm} B 46/7.1. \quad T = \begin{bmatrix} 10 & -5 & 3 \\ 0 & 8 & 6 \\ 0 & -9 & 7 \end{bmatrix} \hspace{1cm} D = \begin{bmatrix} 10 & -5 & 3 \\ 0 & 8 & 6 \\ -1 & 3 & 6 \end{bmatrix} \]

B 53/7.1 \begin{bmatrix} -1 & 0 & 1 \\ 2 & 3 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}

B 3/7.2. The set \( S = \left\{ \begin{bmatrix} t \\ 0 \end{bmatrix} \mid t > 0 \right\} \) cannot be a solution of a linear system, because \( S \) is only a half-line, not a full line or a point.

~1~
8 7/2. \( \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} \) + \( t \begin{bmatrix} 2 \\ -1 \end{bmatrix} \)

Since this represents a line in \( \mathbb{R}^2 \), one would expect to find a linear system having \( \mathbf{y} \) as solution set. Indeed, if \( \mathbf{y} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \), then

\[
\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 2 \\ -3 \end{bmatrix}, \quad \text{hence}
\]

\[
x_1 = 2t \\
x_2 = 1 - 3t
\]

We have \( t = x_1 / 2 \) and \( x_2 = 1 - 3 x_1 / 2 \), or

\[
\frac{3x_1}{2} + x_2 = 1, \quad \text{or} \quad 3x_1 + 2x_2 = 2
\]

The linear equation \( 3x_1 + 2x_2 = 2 \) has \( \mathbf{y} \) as the general solution.

B 5/7.3 Apply the row reduction procedure:

\[
\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} \quad \text{add \[ \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} \] to \[ \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} \] \text{row echelon form}
\]

\[
2x - 3y = 0 \\
\frac{5}{2}y = 2 \\
\quad \quad \rightarrow \quad y = \frac{2}{5}, \quad x = \frac{3y}{2} = -\frac{6}{5}
\]

\[
\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6/5 \\ -4/5 \end{bmatrix}
\]

B 13/7.3

\[
\begin{bmatrix} 4 \\ 7 \\ 5 \\ 18 \\ -2 \\ 1 \\ -1 \\ 0 \end{bmatrix} \quad \text{add \[ \begin{bmatrix} 2 \\ 7 \\ 5 \\ 18 \end{bmatrix} \] to \[ \begin{bmatrix} 0 \\ 2 \\ \frac{3}{2} \\ 9 \end{bmatrix} \] row echelon form}
\]

\[
4x + 7y + 5z = 18 \\
\frac{9}{2}y + \frac{3}{2}z = 9 \\
\quad \quad \rightarrow \quad y = \frac{9 - \frac{3}{2}t}{2}, \quad z = 2 - \frac{1}{3}t
\]

\( z - \text{free variable (= t)} \)
\[
x = \frac{18 - 5t - 7y}{4} = \frac{18 - 5t - 7 \cdot (2 - \frac{2}{3} t)}{4} = \frac{4 - \frac{8}{3} t}{4} = 1 - \frac{2}{3} t
\]

Hence
\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
1 - \frac{2}{3} t \\
2 - \frac{4}{3} t \\
\frac{1}{3}
\end{bmatrix} = \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} + t \begin{bmatrix}
\frac{2}{3} \\
1 \\
1
\end{bmatrix}
\]

\[
\begin{bmatrix}
-12 & 12 & -8 & -8 \\
-16 & 16 & -10 & -10 \\
-3 & 3 & -1 & -1
\end{bmatrix} \begin{align*}
\text{R}_1 & \leftrightarrow \text{R}_3 \\
\sim & \begin{bmatrix}
-3 & 3 & -1 & -1 \\
-16 & 16 & -10 & -10 \\
-12 & 12 & -8 & -8
\end{bmatrix}
\end{align*}
\]

add \(-\frac{16}{3} \text{R}_1 \) to \(\text{R}_2
\]

\[
\begin{bmatrix}
-3 & 3 & -1 & -1 \\
0 & 0 & -\frac{14}{3} & -\frac{14}{3} \\
0 & 0 & -4 & -4
\end{bmatrix} \begin{align*}
\sim & \begin{bmatrix}
-3 & 3 & -1 & -1 \\
0 & 0 & -\frac{14}{3} & -\frac{14}{3} \\
0 & 0 & 0 & 0
\end{bmatrix}
\end{align*}
\]

\[-3x_1 + 3x_2 - x_3 = -1 \quad x_2 \text{ free variable}
\]

\[-\frac{14}{3} x_3 = -\frac{14}{3}
\]

\[-x_2 = t
\]

\[x_3 = 1 \quad \text{and} \quad x_1 = \frac{-1 + x_3 - 3x_2}{-3} = \frac{-1 + 1 - 3t}{-3} = t
\]

Hence
\[
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} = \begin{bmatrix}
t \\
t \\
1
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} + t \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
-4 & 10 & 4 & -8 & 6 \\
0 & 4 & 6 & -7 & 4
\end{bmatrix} \begin{align*}
\text{Already in row echelon form}
\end{align*}
\]

\[-4x_1 + 10x_2 + 4x_3 - 8x_4 = c \quad \Rightarrow \begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix} = \begin{bmatrix}
1 \\
0 \\
0 \\
0
\end{bmatrix} + s \begin{bmatrix}
-11/4 \\
-3/2 \\
1 \\
0
\end{bmatrix} + t \begin{bmatrix}
19/8 \\
7/4 \\
0 \\
0
\end{bmatrix}
\]

\[x_3, x_4 \text{ free variables (} x_3 = s, x_4 = t \) \]

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