1. For each of the following conditions, decide if there is a non-constant holomorphic function defined on the whole complex plane with the given property.
   (a) \( \text{Re} f(z) > 0 \) for all \( z \).
   (b) \( |f(z)| \leq (1 + |z|) / \log(1 + |z|) \) for all \( z \).
   (c) The function \( f \) has 0 and \( \infty \) as its only asymptotic values. (The extended complex number \( \alpha \) is an asymptotic value of \( f \) if there is an unbounded path \( \gamma \) such that \( \lim_{z \to \infty, z \in \gamma} f(z) = \alpha \).

2. Let \( g(z) \) be analytic in the right half-plane \( \{ z \mid \text{Re} z > 0 \} \) with \( |g(z)| < 1 \) for all such \( z \). If \( g(1) = 0 \), how large can \( g(2) \) be?

3. Suppose that \( f(z) \) is a non-constant holomorphic function on a connected open set \( U \subset \mathbb{C} \). Suppose that \( V \) is an open set, that its closure \( \overline{V} \) is a compact subset of \( U \), and that \( |f(z)| \) is constant on the boundary of \( V \). Show that \( f \) has at least one zero in \( V \).

4. Let \( f \) be a complex valued function in the open unit disk \( D \), of the complex plane, such that the functions \( g = f^2 \) and \( h = f^3 \) are both analytic. Prove that \( f \) is analytic in \( D \).

5. Let \( f, g_1, g_2, \ldots \) be entire functions. Assume that the \( k \)th derivatives at 0 satisfy
   (a) \( |g_n^{(k)}(0)| \leq |f^{(k)}(0)| \) for all \( n \) and \( k \). 
   (b) \( \lim_{n \to \infty} g_n^{(k)}(0) \) exists for all \( k \).
   Prove that the sequence \( \{g_n\} \) converges uniformly on compact subsets and that its limit is an entire function.

6. Suppose that \( f(z) \) is analytic and satisfies \( f(1/z) = f(z) \) for all \( z \in \mathbb{C} \setminus \{0\} \).
   (a) Write down the general Laurent expansion for \( f \).
   (b) Show that the coefficients of this expansion are all real if this \( f \) has real values on the unit circle, \( |z| = 1 \).