Welcome to Math 211
Math 211 Section 3 – John C. Polking

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Ordinary Differential Equations with Linear Algebra
There are four themes to the course:
• Applications & modeling.
  • Mechanics, electric circuits, population genetics epidemiology, pollution, pharmacology, personal finance, etc.
• Analytic solutions.
  • Solutions which are given by an explicit formula.
• Numerical solutions.
  • Approximate solutions computed at a discrete set of points.
• Qualitative analysis.
  • Properties of solutions without knowing a formula for the solution.

Math 211 Web Pages
• Official source of information about the course.
  http://www.owlnet.rice.edu/~math211/.
• Source for the slides for section 3.
  http://math.rice.edu/~polking/slidesf01.html.

What Is a Derivative?
• The rate of change of a function.
• The slope of the tangent line to the graph of a function.
• The best linear approximation to the function.
• The limit of difference quotients.
• Rules and tables that allow computation.
What Is an Integral?
- The area under the graph of a function.
- An anti-derivative.
- Rules and tables for computing.

Differential Equations
An equation involving an unknown function and one or more of its derivatives, in addition to the independent variable.
- Example: $y' = 2ty$
- General equation: $y' = f(t, y)$
- $t$ is the independent variable.
- $y = y(t)$ is the unknown function.
- $y' = 2ty$ is of order 1.

Equations and Solutions
$y' = f(t, y) \quad y' = 2ty$
A solution is a function $y(t)$, defined for $t$ in an interval, which is differentiable at each point and satisfies $y'(t) = f(t, y(t))$ for every point $t$ in the interval.
- What is a function?
- An ODE is a function generator.
Example: \( y' = 2ty \)

Claim: \( y(t) = e^{t^2} \) is a solution.

- Verify by substitution.
  - Left-hand side: \( y'(t) = 2te^{t^2} \)
  - Right-hand side: \( 2ty(t) = 2te^{t^2} \)
- Therefore \( y'(t) = 2ty(t) \), if \( y(t) = e^{t^2} \).
- Verification by substitution is always available.

Is \( y(t) = e^t \) a solution to the equation \( y' = 2ty \)?

- Check by substitution.
  - Left-hand side: \( y'(t) = e^t \)
  - Right-hand side: \( 2ty(t) = 2te^t \)
- Therefore \( y'(t) \neq 2ty(t) \), if \( y(t) = e^t \).
- \( y(t) = e^t \) is not a solution to the equation \( y' = 2ty \).

Types of Solutions

For the equation \( y' = 2ty \)

- \( y(t) = \frac{1}{2}e^{t^2} \) is a solution. It is a particular solution.
- \( y(t) = Ce^{t^2} \) is a solution for any constant \( C \). This is a general solution.

General solutions contain arbitrary constants. Particular solutions do not.
**Initial Value Problem (IVP)**

A differential equation & an initial condition.

- Example: \( y' = -2ty \) with \( y(0) = 4 \).
- General solution: \( y(t) = Ce^{-t^2} \).
- Plug in the initial condition:
  \[
  y(0) = 4, \\
  Ce^0 = 4, \\
  C = 4
  \]

Solution to the IVP: \( y(t) = 4e^{-t^2} \).

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**Normal Form of an Equation**

\[ y' = f(t, y) \]

Example: \((1 + t^2)y' + y^3 = t^3\)

- This equation is not in normal form.
- Solve for \( y' \) to put into normal form:
  \[
  y' = \frac{t^3 - y^2}{1 + t^2}
  \]