$$|D| = 5(-3) - (-2)6 = -3 \quad D^{-1} = \begin{bmatrix} \frac{-2}{3} & \frac{-6}{3} \\ \frac{2}{3} & \frac{5}{3} \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -2/3 & -5/3 \end{bmatrix}.$$ 

2.68 Suppose $A$ is a square matrix. Show: (a) $A + A^T$ is symmetric
(b) $A - A^T$ is skew-symmetric, (c) $A = B + C$, where $B$ is symmetric and $C$ is skew-symmetric.

**Proof.**
(a) $(A + A^T)^T = A^T + A = A + A^T$ thus symmetric
(b) $(A - A^T)^T = A^T - A = -(A - A^T)$ thus skew-symmetric
(c) $A = \frac{(A + A^T) + (A - A^T)}{2} = \frac{A + A^T}{2} + \frac{A - A^T}{2}$

Let $B = \frac{A + A^T}{2}, C = \frac{A - A^T}{2}$ $A = B + C$, $B$ is symmetric and $C$ is skew-symmetric.

2.70 Suppose $A$ and $B$ are symmetric. Show that the following are also symmetric:

(a) $A + B$
(b) $kA$, for any scalar $k$
(c) $A^2$
(d) $A^n$, for $n > 0$
(e) $f(A)$, for any polynomial $f(x)$

**Proof.**
(a) $(A + B)^T = A^T + B^T = (A + B)$
(b) $(kA)^T = kA^T = (kA)$
(c) $(A^2)^T = (AA)^T = A^T A = AA = (A^2)$
(d) $(A^n)^T = (\underbrace{A \cdot \cdots \cdot A}_n)^T = A^T A^T \cdots A^T = A \cdots A = (A^n)$
(e) by (a), (b), and (d) we know since $f(A)$ is the linear combination of $I, A, A^2, \ldots, A^n$, $\ldots$, $f(A)$ is symmetric.