# MATH 466/566 SPRING 2024

CHELSEA WALTON RICE U.

#### LAST TIME

LECTURE#9

- · FUNCTORS
- · BIFUNCTORS & MULTIFUNCTORS
- · NATURAL TRANSFORMATIONS
- · COMPOSITIONS OF NATURAL TRANSFORMATIONS

# TOPICS:

I. ISOMORPHISM OF CATEGORIES (§2.4.1)

II. EQUIVALENCE OF CATEGORIES (882.4.2-2.4.3)

III. MORITA EQUIVALENCE (§2.4.3)

# A CATEGORY &

CONSISTS OF:

- (a) OBJECTS.
- (b) MORPHISMS HOME(X,Y) YX,Y EC.
- (c)  $id_X: X \rightarrow X$  $\forall x \in \mathcal{C}$ .
- 3:X→Y. A t:M→X (Y) Dt:M→A

SATISFYING

Associativity (hg)f = h(gf)

UNITALITY

idx f = f, gidx = g

- A CATEGORY & CONSISTS OF:
- (a) OBJECTS.
- (b) MORPHISMS Home(x,y) YX,Y EC.
- (c)  $id_X:X \rightarrow X$ YXEC.

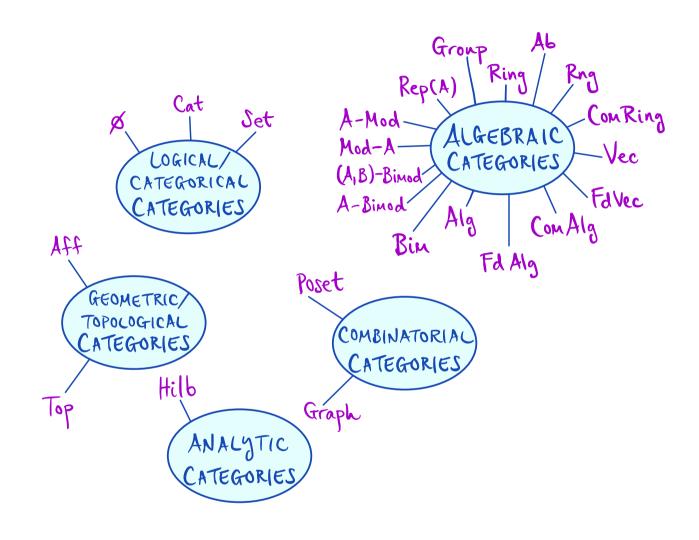
SATISFYING

ASSOCIATIVITY

(hg)f = h(gf)

UNITALITY

idxf=f, gidx=g



### WHEN ARE TWO CATEGORIES CONSIDERED THE SAME?

A CATEGORY & CONSISTS OF:

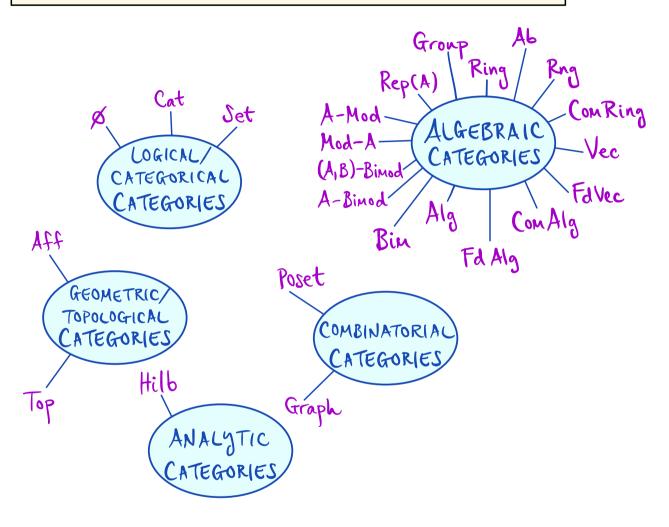
- (a) OBJECTS.
- (b) MORPHISMS Home(x,y) YX,Y EG.
- (c)  $id_X:X \rightarrow X$ YXEC.
- $9 \cdot \lambda \rightarrow \gamma$ .

SATISFYING

ASSOCIATIVITY

(hg)f = h(gf)

UNITALITY idx f = f, gidx = g



# A CATEGORY & CONSISTS OF:

- (a) OBJECTS.
- (b) MORPHISMS Home(x,y) YX,Y EG.
- (c)  $id_X:X \rightarrow X$ YXEC.
- $9 \cdot \lambda \rightarrow \gamma$ .

SATISFYING

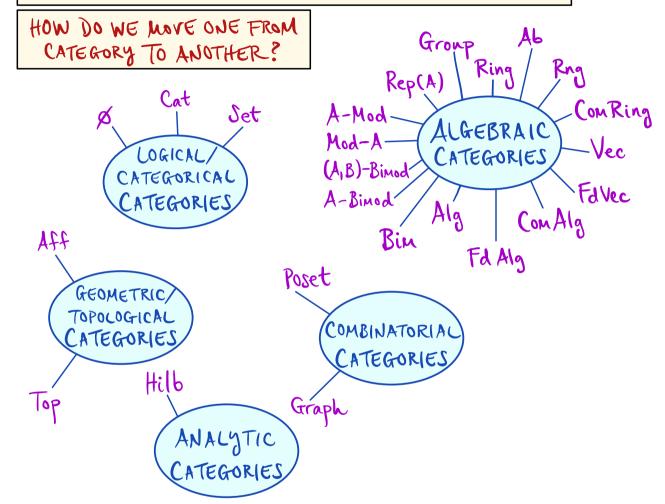
ASSOCIATIVITY

(hg)f = h(gf)

UNITALITY

idx f = f, gidx = g

WHEN ARE TWO CATEGORIES CONSIDERED THE SAME?



WHEN ARE TWO CATEGORIES CONSIDERED THE SAME?

A FUNCTOR

V F: C -> B

(RESP.,

CONTRAVARIANT)

CONSISTS OF:

(a) F(x) & Q & X & &

(b) F(g): F(X) → F(Y) ∈ B

RESP.,

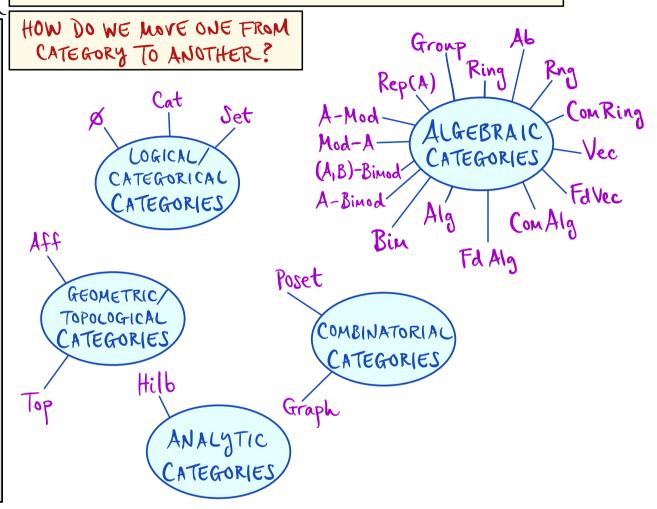
 $F(g): F(Y) \rightarrow F(X) \in \emptyset$ 

¥g:x→y∈e.

RESPECTING:

IDENTITY \$

COMPOSED MORPHISMS



WHEN ARE TWO CATEGORIES CONSIDERED THE SAME?

A FUNCTOR

V F: 6 -> 8

(RESP.,

CONTRAVARIANT)

CONSISTS OF:

(a) F(x) & D YX & C.

(b) F(g): F(X) → F(Y) ∈ B

RESP.,

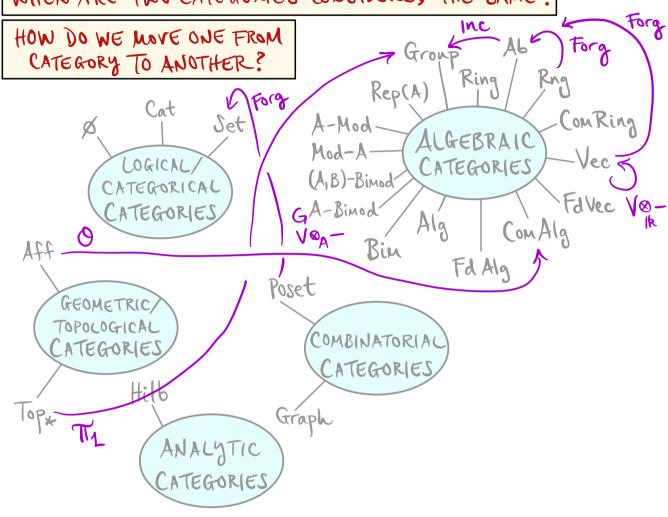
 $F(g): F(Y) \rightarrow F(X) \in \emptyset$ 

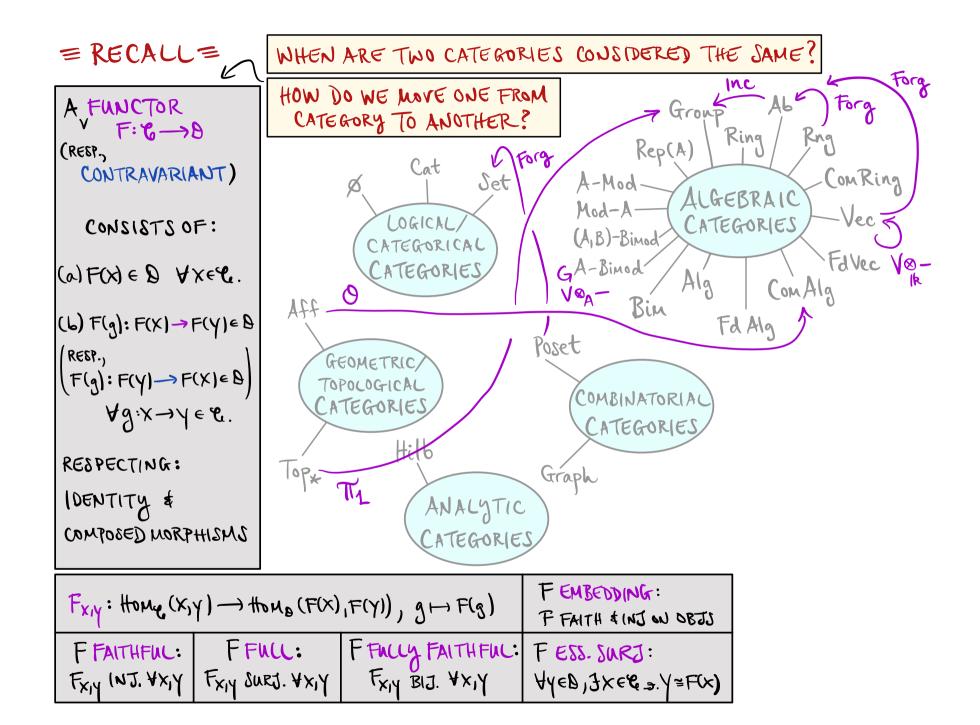
¥g:x→y∈e.

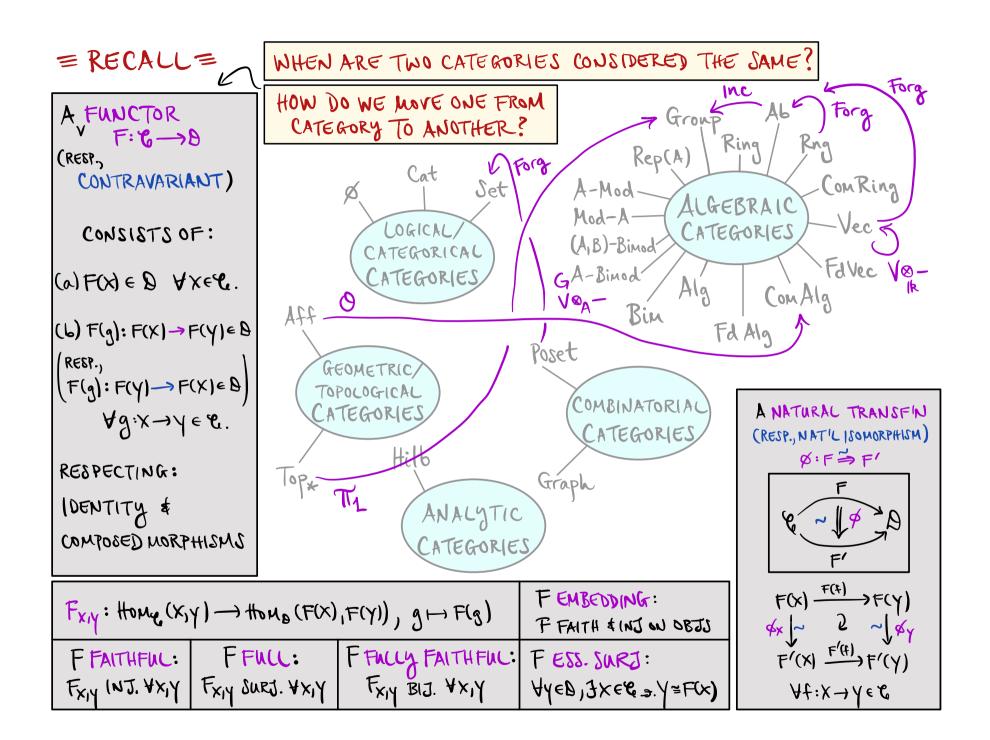
RESPECTING:

IDENTITY \$

COMPOSED MORPHISMS







WHEN ARE TWO CATEGORIES CONSIDERED THE SAME?

NOW SOME ANSWERS

6, & CATEGORIES

& and &

ARE SAID TO BE

ISOMORPHIC

IF J FUNCTORS

F:6-10 \$ G:0-7

SUCH THAT

GF = Ide

& FG = Ido

WRITE &= B

6, & CATEGORIES

& AND B

ARE SAID TO BE

ISOMORPHIC

IF FINCTORS

F: C→D \$ G:D→C

GF = Ide

SUCH THAT

& FG = Ido

WRITE &= B

EXAMPLE: G = GROUP

GET: G-Mod = Rep(G)

& and & ARE SAID TO BE ISOMORPHIC IF 3 FUNCTORS F: 6-0 \$ G:0-6 SUCH THAT GF = Ide & FG = IdA WRITE &= B

& and & ARE SAID TO BE ISOMORPHIC IF 3 FUNCTORS F: 6-9 \$ 6:0-6 SUCH THAT GF = Ide & FG = IdA WRITE &= B

EXAMPLE: 
$$G = GRONP$$

GET:  $G-Mod \cong Rep(G)$ 
 $F: G-Mod \longrightarrow Rep(G)$ 
 $(V, D: G \times V \to V) \mapsto (V, pv: G \to GL(V))$ 

Vec

 $(gh) Dv = gD(NDV)$ 
 $p(gh)(v) = gh Dv = gD(NDV)$ 
 $p(g)p(h)(v) = gD(p(h)(v))$ 
 $p(g)p(h)(v) = gD(p(h)(v))$ 
 $p(g, V) \mapsto p(g, V) \mapsto p(g, V)$ 
 $p(g, V) \mapsto p(g, V) \mapsto p(g, V)$ 

Vec

& and & ARE SAID TO BE ISOMORPHIC IF 3 FUNCTORS F: 6-9 \$ 6:0-6 SUCH THAT GF = Ide & FG = IdA WRITE &= B

EXAMPLE: G = GROWP GET: G-Mod = Rep (G) F: G-Mod - Rep(G) p(gh)(v) = (gh) bv = gb(hbv)D(a)p(h)(a) = 3p(b(h)(a))F': Rep(G) ----- G-Mod  $(V, p: G \longrightarrow GUV)) \longmapsto (V, D_V: G \times V \longrightarrow V)$   $(q, \sigma) \mapsto \rho_{\mathcal{D}}(\sigma)$ CHECK F'F = Id fr-mod & FF' = Id Rep(G)

6, & CATEGORIES

& and & ARE SAID TO BE ISOMORPHIC IF 3 FUNCTORS F: 6-9 \$ 6:0-6 SUCH THAT GF = Ide & FG = IdA WRITE &= B

UPGRADE OF EXAMPLE: G = GROWP EXER. 1.13 GET: G-Mod = Rep (G) F: G-Mod - Rep(G) p(gh)(v) = (gh) Dv = go(hov)  $\rho(g)\rho(h)(v) = g \Rightarrow (\rho(h)(v))$ F': Rep(G) ---- G-Mod  $(V, p: G \longrightarrow GUV)) \longmapsto (V, D_V: G \times V \longrightarrow V)$   $(q, \sigma) \mapsto \rho_{\mathcal{D}}(\sigma)$ CHECK F'F = Id fr-mod & FF' = Id Rep(G)

E, & CATEGORIES

& and & ARE SAID TO BE ISOMORPHIC IF 3 FUNCTORS F: 6-10 \$ G:0-6 SUCH THAT GF = Ide & FG = IdA WRITE &= B

EXERCISE 2.30: SHOW-G-Mod = Rep(G) = Rep(IKG) = IKG-Mod

UPGRADE OF EXAMPLE: G = GROWP EXER. 1.13 GET: G-Mod = Rep (G) F: G-Mod - Rep(G) p(gh)(v) = (gh) bv = gb(hbv) $\rho(g)\rho(h)(v) = g \Rightarrow (\rho(h)(v))$ F': Rep(G) ---- G-Mod  $(V, p: G \longrightarrow GUV) \longrightarrow (V, D_V: G \times V \longrightarrow V)$   $(g, \sigma) \mapsto \rho_{g}(\sigma)$ CHECK F'F = Id fr-mod & FF' = Id Rep(G)

6, & CATEGORIES

& and & ARE SAID TO BE ISOMORPHIC IF 3 FUNCTORS F: 6-10 \$ G:0-6 SUCH THAT GF = Ide & FG = IdA WRITE &= B

EXERCISE 2.30: SHOW
G-Mod = Rep(G)

= Rep(IkG) = (V, p: |kG-) End|(V))

= IkG-Mod = (V, p: |kG x V -) V)

UPGRADE OF EXAMPLE: G = GROWP EXER. 1.13 GET: G-Mod = Rep (G) F: G-Mod - Rep(G) p(gh)(v) = (gh) bv = gb(hbv) $\rho(g)\rho(h)(v) = gp(\rho(h)(v))$ F': Rep(G) ----- G-Mod  $(V, p: G \rightarrow GUV) \mapsto (V, D_V: G \times V \rightarrow V)$   $(q, \sigma) \mapsto \rho_{2}(\sigma)$ CHECK F'F = Id f-mod & FF' = Id Rep(G)

6, & CATEGORIES

C AND B

ARE ISOMORPHIC IF  $JF: C \rightarrow D + G: D \rightarrow C$  .3. GF = Ide + FG = IdeWRITE C = D

CONSIDER FOLLSWECATEGORY OF FOLLOW TAKE & = FULL SWECATEGORY OF FOLLOW ON OBJECTS & RONG NEW PERHAPS FOLLOW AND THE "SAME" AS EVERY F.D. VECTOR SPACE IS = 12 PM FOR SOME NEW.

C AND B

ARE ISOMORPHIC IF  $JF: C \rightarrow D + G: D \rightarrow C$ .3.  $GF = Id_C + FG = Id_D$ WRITE C = D

CONSIDER Faller / IR FIELD TAKE & = FULL SUBCATEGORY OF FLVECK ON OBJECTS } RON JNEN PERHAPS Follow & & ARE THE "SAME" AS EVERY F.D. VECTOR SPACE IS = 12 PAR FOR SOME NEW. F: FdVec -> & G: & -> FdVec V -> R DainRV R DAINRV

C AND B

ARE ISOMORPHIC IF  $JF: C \rightarrow D + G: D \rightarrow C$  .3.  $GF = Td_C + FG = Td_D$ WRITE C = D

CONSIDER Faller / IR FIELD TAKE & = FULL SUBCATEGORY OF FLVECK ON OBJECTS } 12mm I NEW PERHAPS Foller & & ARE THE "SAME" AS EVERY F.D. VECTOR SPACE IS = 12 PAR FOR SOME NEW. TRY: F: FdVec -> & & G: & -> FdVec V -> 1k DdinkV 1k Dn HERE,  $FG(\mathbb{R}^{\oplus n}) = F(\mathbb{R}^{\oplus n}) = \mathbb{R}^{\oplus n}$ . BUT GF(V) = G(K DdimkV) = K DdimkV

C AND B

ARE ISOMORPHIC IF  $JF:C \rightarrow D + G:D \rightarrow C$  ... GF = Ide + FG = IdeWRITE C = D

CONSIDER Faller / IR FIELD TAKE & = FULL SUBCATEGORY OF FLVECK ON OBJECTS } 12mm I NEW PERHAPS Favec & & ARE THE "SAME" AS EVERY F.D. VECTOR SPACE IS = 12 PM FOR SOME TRY: F: FdVec -> & & G: & -> FdVec HERE, FG(ken) = F(ken) = ken. BUT GF(V) = G( | Ddin | V) = | Ddin | V : FdVec & J

C AND B

ARE ISOMORPHIC IF  $JF:C \rightarrow D + G:D \rightarrow C$  ... GF = Ide + FG = IdeWRITE C = D

CONSIDER Faller / IR FIELD TAKE & = FULL SUBCATEGORY OF FLVECK ON OBJECTS } 12mm I NEW PERHAPS Follow & & ARE THE "SAME" AS EVERY F.D. VECTOR SPACE IS = 12 PAR FOR SOME NEW TRY: F: FdVec -> & & G: & -> FdVec HERE, FG(ken) = F(ken) = ken. BUT GF(V) = G(1k Ddin1RV) = 1k Ddin1RV + V : FdVec & J WILL WEAKEN NOTION OF "SAMENESS"...

C AND B

ARE ISOMORPHIC IF  $JF: C \rightarrow D + G: D \rightarrow C$  ...  $GF = Td_C + FG = Td_D$ WRITE C = D

SKELETON OF G = FULL SUBCATEGORY Skel(C) OF Y ON ISOCLASSES OF Obj(C)

CONSIDER FAVEC / IR FIELD TAKE & = FULL SUBCATEGORY OF FLVECK ON OBJECTS } 12mm I NEW PERHAPS Follow & & ARE THE "SAME" AS EVERY F.D. VECTOR SPACE IS = 12 PM FOR SOME NEW. TRY: F: FdVec -> & & G: & -> FdVec HERE, FG(ken) = F(ken) = ken. BUT GF(V) = G(K DdinkV) = K DdinkV + V :. FdVec & J WILL WEAKEN NOTION OF "SAMENESS"...

C AND B

ARE ISOMORPHIC IF  $JF: C \rightarrow D + G: D \rightarrow C$  .3.  $GF = Td_C + FG = Td_D$ WRITE C = D

SKELETON OF C = FULL SUBCATEGORY Skel(C) OF C ON ISOCLASSES OF Obj(C)

CONSIDER FAVEC / IR FIELD TAKE & = FULL SUBCATEGORY OF FAVECIE ON OBJECTS } 12mm I NEW PERHAPS Follow & X ARE THE "SAME" AS EVERY F.D. VECTOR SPACE IS = 12 PM FOR SOME NEW F: FdVec -> & G: & -> FdVec V -> 1/2 dingv HERE, FG(ken) = F(ken) = ken. BUT GF(V) = G(K DdinkV) = K DdinkV + V : FdVec & 1 WILL WEAKEN NOTION OF "SAMENESS"...

C AND B

ARE ISOMORPHIC IF  $JF: C \rightarrow D + G: D \rightarrow C$  .3. GF = Ide + FG = IdeWRITE C = D

SKELETON OF C = FULL SUBCATEGORY Skel(C) OF C ON ISOCLASSES OF Obj(C)

CONSIDER FAVEC / IR FIELD TAKE & = FULL SUBCATEGORY OF FAVECIE Skel (FdVec) ON OBJECTS [ 12m Jnen PERHAPS Foller & X ARE THE "SAME" AS EVERY F.D. VECTOR SPACE IS = 12 PM FOR SOME NEW F: FdVec -> & G: & -> FdVec V -> 1/2 Ddingv HERE, FG(ken) = F(ken) = ken. BUT GF(V) = G(R DdingeV) = R DdingeV = V : FdVec & 1 WILL WEAKEN NOTION OF "SAMENESS"...

C AND B

ARE ISOMORPHIC IF  $JF:C \rightarrow D + G:D \rightarrow C$  ... GF = Ide + FG = IdeWRITE C = D

SKELETON OF G

= FULL SUBCATEGORY

Ske((C) OF G

ON ISOCLASSES OF Obj(C)

EXER. 2.33
Skel(2)=2 \$\ightarrow\$ Skel(2)=2

CONSIDER FAVEC / IR FIELD TAKE & = FULL SUBCATEGORY OF FAVECIE Skel (FdVec) ON OBJECTS [ 12m Jnen PERHAPS Foller & X ARE THE "SAME" AS EVERY F.D. VECTOR SPACE IS = 12 PM FOR SOME NEW. TRY: F: FdVec -> & & G: & -> FdVec V -> 12 Ddingv HERE, FG(ken) = F(ken) = ken. BUT GF(V) = G(IR DdimIRV) = IR DdimIRV + V : FdVec & 1 WILL WEAKEN NOTION OF "SAMENESS"...

C AND B

ARE ISOMORPHIC IF  $JF:C \rightarrow D + G:D \rightarrow C$  ... GF = Ide + FG = IdeWRITE C = D

SKELETON OF G

= FULL SUBCATEGORY

Ske((C)) OF G

ON ISOCLASSES OF Obj(C)

EXER. 2.33  $Skel(\mathcal{L}) \cong \mathcal{L} \iff Skel(\mathcal{L}) = \mathcal{L}$ 

CONSIDER FAVEC / IR FIELD TAKE & = FULL SUBCATEGORY OF FAVECIE Skel (FdVec) ON OBJECTS & IRON JNEN PERHAPS Foller & X ARE THE "SAME" AS EVERY F.D. VECTOR SPACE IS = 12 PM FOR SOME NEW F: FdVec -> & G: & -> FdVec V -> 12 Ddingv HERE, FG(ken) = F(ken) = ken. BUT GF(V) = G(K DdiMRV) = K DdiMRV + V :. FdVec & J WILL WEAKEN NOTION OF "SAMENESS"... EQUIV. C AND B

ARE ISOMORPHIC IF  $JF:C\rightarrow D + G:D\rightarrow C$  .3. GF=Ide + FG=IdeWRITE C=D

SKELETON OF G = FULL SUBCATEGORY Skel(C) OF G ON ISOCLASSES OF Obj(C)

EXER. 2.33

Skel(t)=t \iff Skel(t)=t

CONSIDER Fallec / IR FIELD TAKE & = FULL SUBCATEGORY OF FAVECIE Skel (FdVec) ON OBJECTS [RON] NEN F: FdVec -> & & G: & -> FdVec V -> 12 DainkV HERE, FG(ken) = F(ken) = ken. BUT GF(V) = G(IR DdimIRV) = IR DdimIRV + V :. FdVec & 1 WILL WEAKEN NOTION OF "SAMENESS"... EQUIV. C AND B

ARE EQUIVALENT IF  $f: C \rightarrow D + G: D \rightarrow C$   $f: C \rightarrow D + G: D \rightarrow C$   $f: C \rightarrow D + G: D \rightarrow C$ AF = Ide + FG = Ide

WRITE C = D

SKELETON OF G = FULL SUBCATEGORY Skel(C) OF G ON ISOCLASSES OF Obj(C)

EXER. 2.33  $Skel(\mathcal{L}) \cong \mathcal{L} \iff Skel(\mathcal{L}) = \mathcal{L}$ 

CONSIDER Fallec / IR FIELD TAKE & = FULL SUBCATEGORY OF FAVECIE Skel (FdVec) ON OBJECTS [ 12m Jnen F: FdVec -> & & G: & -> FdVec V -> 12 DainkV HERE, FG(ken) = F(ken) = ken. BUT GF(V) = G(IR DdimIRV) = IR DdimIRV + V :. FdVec & 1 WILL WEAKEN NOTION OF "SAMENESS"... EQUIV.

6, B CATEGORIES

C AND B

ARE EQUIVALENT IF  $f: C \rightarrow 0 + G: 0 \rightarrow C$   $f: C \rightarrow 0 + G: 0 \rightarrow C$   $G: C \rightarrow C$   $G: C \rightarrow C$ WRITE C = CWRITE C = C

SKELETON OF G = FULL SUBCATEGORY Skel(C) OF G ON ISOCLASSES OF Obj(C)

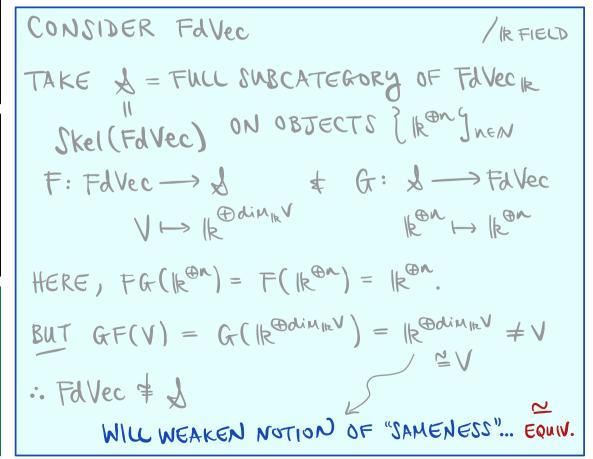
EXER. 2.33

Skel(t)=t \iff Skel(t)=t

JNATURAL
ISOMORPHISMS: & Ing & DING

G="QNASI-INVERSE" Ide Ide

OF F



E, B CATEGORIES

B DIA & BRE EQUIVALENT IF

F: C→O \$ G:O→C

GF=Ide \$ FG=Ide

WRITE C=O

SKELETON OF G = FULL SUBCATEGORY Skel(&) OF G ON ISOCLASSES OF Obj(G)

EXER. 2.33

Skel(t)= t \iff Skel(t)= t

JNATURAL
ISOMORPHISMS: & Ing. & D. Ing.

Tide

Tide

Tide

CONSIDER FAVEC / IR FIELD TAKE & = FULL SUBCATEGORY OF FLVEC IR Skel (FdVec) ON OBJECTS & IRON JNEN F: FdVec -> & # G: & -> FdVec HERE, FG(ken) = F(ken) = ken. BUT GF(V) = G(R DdinkV) = R DdinkV +V :. FdVec & 1

E, B CATEGORIES

B CHA B

ARE EQUIVALENT IF

F: C→O \$ G:O→C

GF = Ide \$ FG = Ide

WRITE C = O

SKELETON OF G = FULL SUBCATEGORY Skel(C) OF G ON ISOCLASSES OF Obj(C)

EXER. 2.33

Skel(t)=t \iff Skel(t)=t

JNATURAL
ISOMORPHISMS: & Interest of the Table

The Table Table

CONSIDER FAVEC / IR FIELD TAKE & = FULL SUBCATEGORY OF FLVEC IR Skel (FdVec) ON OBJECTS & IRON JNEN F: FdVec -> & & G: & -> FdVec V -> 1/2 dinkV 1/2 1/2 1/2 1/2 HERE, FG(ken) = F(ken) = ken. BUT GF(V) = G(IR OdinIRV) = IR OdinIRV + V :. FdVec & 1

E, B CATEGORIES

C AND B

ARE EQUIVALENT IF  $3F:C \rightarrow D \neq G:D \rightarrow C$   $3F:C \rightarrow C \neq G:D \rightarrow C$ GF=Ide  $\neq FG=Ide$ WRITE C = D

SKELETON OF G

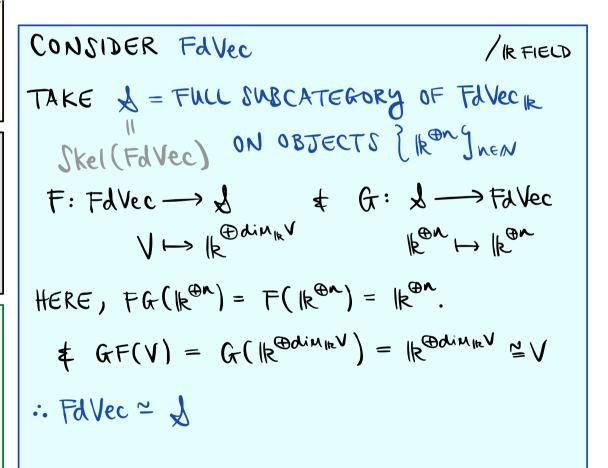
= FULL SUBCATEGORY

Skel(C) OF G

ON ISOCLASSES OF Obj(C)

EXER. 2.33

Skel(t)=t \iff Skel(t)=t



6, & CATEGORIES

G AND B

ARE EQUIVALENT IF

FF: C→B & G: B→C

GF = Ide & FG = Ide

WRITE C = B

SKELETON OF C = FULL SUBCATEGORY Ske((C)) OF C ON ISOCLASSES OF Obj(C)

EXER. 2.33

Skel(2)=2 \ Skel(2)=2

JNATURAL 150MORPHISMS: & Ing. # 150 Ing.

Ide Ide Ide

CONSIDER FAVEC / IR FIELD TAKE & = FULL SUBCATEGORY OF FAVECIE Skel (FdVec) ON OBJECTS & IRON JNEN F: FdVec -> & & G: & -> FdVec V -> 1/2 dinkV 1/2 1/2 1/2 1/2 HERE, FG(ken) = F(ken) = ken. & GF(V) = G( IR BdinkV) = IR BdinkV WV :. FdVec ~ 1

E, B CATEGORIES

C AND B

ARE EQUIVALENT IF  $3F:C\rightarrow 0 + G:0\rightarrow c$  3F:Tde + FG=TdeWRITE C=0

SKELETON OF G = FULL SUBCATEGORY Skel(C) OF G ON ISOCLASSES OF Obj(C)

EXER. 2.33

Skel( $\mathcal{C}$ ) =  $\mathcal{C}$   $\Leftrightarrow$  Skel( $\mathcal{C}$ ) =  $\mathcal{C}$ Skel( $\mathcal{C}$ ) =  $\mathcal{C}$  ALWAYS  $\mathcal{C} = \mathcal{D} \Leftrightarrow Skel(\mathcal{C}) = Skel(\mathcal{D})$ 

JNATURAL
ISOMORPHISMS: & Ing. & DING.

Ide Ide

CONSIDER Faller / IR FIELD TAKE & = FULL SUBCATEGORY OF FAVECIE Skel (FdVec) ON OBJECTS & IRON JNEN F: FdVec -> & & G: & -> FdVec V -> 1/2 dinkV 1/2 1/2 1/2 1/2 HERE, FG(ken) = F(ken) = ken. & GF(V) = G( IR BdinkV) = IR BdinkV WV :. FdVec = 1

E, B CATEGORIES

C AND B

ARE EQUIVALENT IF  $f: C \rightarrow 0 + G: 0 \rightarrow C$   $f: C \rightarrow 0 + G: 0 \rightarrow C$ GF= Ide + FG= Ide

WRITE C = 0

JNATURAL
ISOMORPHISMS: & Interest to the Table

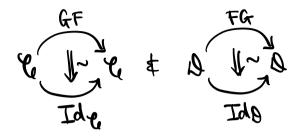
Table

Table

LIKE TWO STRUCTURES ARE THE SAME

E, B CATEGORIES

JNATURAL ISOMORPHISMS:



J MUTUALLY INVERSE STRUCTURE MAPS BETWEEN THEM

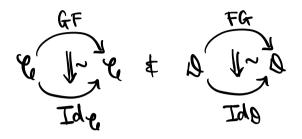


LIKE TWO STRUCTURES ARE THE SAME

E, B CATEGORIES

ARE EQUIVALENT IF  $3F:C\rightarrow 0 \ddagger G:0\rightarrow C$  3  $3F:Z\rightarrow 0$   $3F:Z\rightarrow 0$ WRITE C=0

JNATURAL ISOMORPHISMS:



J MUTUALLY INVERSE STRUCTURE MAPS BETWEEN THEM



LIKE TWO STRUCTURES ARE THE SAME



JA BIJECTIVE STRUCTURE MAP FROM ONE TO THE OTHER

E, & CATEGORIES

& and &

ARE EQUIVALENT IF

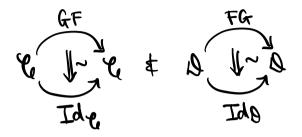
3F:6→0 \$ G:0→6

.<del>)</del>.

GF= Ide & FG= Ide

WRITE 6 = 0

JNATURAL ISOMORPHISMS:



J MUTUALLY INVERSE STRUCTURE MAPS
BETWEEN THEM

A CHARACTERIZATION IN TERMS OF ...

 $F_{X,Y}$ : Home  $(X,Y) \rightarrow \text{Hom}_{\mathcal{B}}(F(X),F(Y))$   $g \mapsto F(g)$ 

F FAITHFUL: FXIY INJ. YXIY

FFULL: Fxiy Surd. 4xiy

FFILLY FAITHFIL: FXIY BIJ. YXIY

FESS. SURJ:

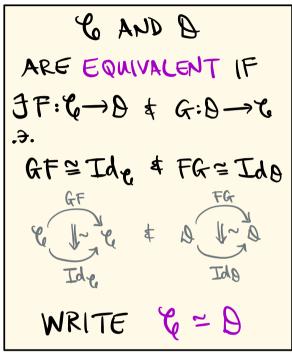
YYED, 3xE& ... Y=F(x)

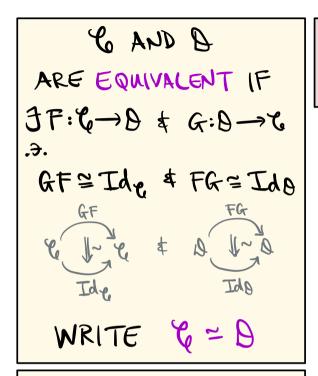
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LIKE TWO STRUCTURES ARE THE SAME



JA BIJECTIVE STRUCTURE MAP FROM ONE TO THE OTHER





F<sub>X,Y</sub>: Hong(X,Y) — Hong(F(X),F(Y))

g \rightarrow F(g)

F FAITHFUL: F<sub>X,Y</sub> INJ. 4X,Y

F FULL: F<sub>X,Y</sub> SURJ. 4X,Y

F FULLy FAITHFUL: F<sub>X,Y</sub> BIJ. 4X,Y

F ESS. SURJ:

44 \( \text{4} \), \( \text{3} \) \( \text{4} \)

Y \( \text{8} \), \( \text{3} \) \( \text{4} \)

CAND B

ARE EQUIVALENT IF

3F: C -> D & G: D -> C

3F: Tde & FG: Ido

FG

WRITE C = D

PF((=>) SAY ∃ Ø: Ide => GF & Y: FG => Ide DEFN

CLAIM: FIS FULLY FAITHFUL AND ESS. SURJ.

FFAITHFUL: Fxiy INJ. YXiY

FFULL: Fxiy SURJ. YXiY

FFULL: FAITHFUL: Fxiy BIJ. YXiY

FFULLY FAITHFUL: Fxiy BIJ. YXiY

FESS. SURJ:

YYEB, JXEE 3. Y=F(X)

CAND B

ARE EQUIVALENT IF

JF: C -> D & G: D -> C

...

GF == Ide & FG == Ide

GF

Tde & Ide

WRITE C == D

PF(=) SAY = Ø: Ide = GF & Y: FG = Ide DEFN

CLAIM: FIS FULLY FAITHFUL AND ESS. SURJ.

TAKE Y & O. THEN FG(Y) = Y = F ESS. SURJ.

Fxiy: Home(x,y) -> Home(F(x),F(y))
g +> F(g)

F FAITHFUL: Fxiy INJ. YXiY

F FULL: Fxiy SURJ. YXiY

F FULLY FAITHFUL: Fxiy BIJ. YXiY

F ESS. SURJ:

YYED, 3XEC 3. Y=F(x)

CAND B

ARE EQUIVALENT IF

JF: C -> D & G: D -> C

...

GF = Ide & FG = Ide

CF

Tde & Ide

WRITE C = D

Fx.y: Home(x,y) — Home(F(x),F(y))

g \rightarrow F(g)

F FAITHFUL: Fx,y (NJ. \forall x,y)

F FULL: Fx,y SURJ. \forall x,y

F FULLY FAITHFUL: Fx,y BIJ. \forall x,y

F ESS. SURJ:

\forall y \in B, \forall x \in B

THEOREM

G=0 

FINCTOR F: 4-0

PF((⇒) SAY ∃ Ø: Idy ≃ GF & Y: FG ⇒ Ido DEFN CLAIM: FIS FULLY FAITHFUL AND ESS. SURJ.

TAKE YEO. THEN FG(Y) => F ESS. SURJ.

NOW  $\forall g: X \rightarrow X' \in \mathcal{C}$ , GET  $X \xrightarrow{g} X'$ THIS IMPLIES FIS FAITHFUL  $\forall_X \models 2 \Rightarrow \bigwedge_{x'} \bigvee_{x'} \downarrow_{x'}$ 

$$\begin{array}{ccc}
X & \xrightarrow{\vartheta} & X' \\
\varphi_{X} \downarrow^{\cong} & 2 & \cong \uparrow \varphi_{X'}^{-1} \\
GF(X) & \xrightarrow{GF(Y)} & GF(X')
\end{array}$$

CAND B

ARE EQUIVALENT IF

JF: C -> D & G: D -> C

...

AF = Ide & FG = Ide

VRITE C - D

WRITE C - D

Fry: Homy (X,Y) -> Homo (F(X), F(Y))

g -> F(g)

F FAITHFUL: Fx,y (NJ. 4X,Y)

F FULL: Fx,y SURJ. 4X,Y

F FULLY FAITHFUL: Fx,y BIJ. 4X,Y

F ESS. SURJ:

Y \in B, 3x \in B, \in F(X)

THEOREM 6=0 = FINITED FILE FUNCTOR F: 4 - 0 PF/(⇒) SAY ∃Ø: Ide ~ JA (€) THE PAZ (€) THE CLAIM: FIS FULLY FAITHFUL AND ESS. SURJ. TAKE YEO. THEN FG(Y) > Y => F ESS. SURJ. NOW 4 2: X -> X' & C , GET X -3 X' THIS IMPLIES FIS FAITHFAL SX = 2 = 1 8/21  $F(g) = F(g) \implies GF(g) = GF(g)$  $\begin{array}{c}
GF(X) \longrightarrow GF(X') \\
GF(3) & GF(3)
\end{array}$ 

CAND B

ARE EQUIVALENT IF

JF: C -> D & G: D -> C

...

GF = Ide & FG = Ide

VRITE C = D

WRITE C = D

F<sub>X,Y</sub>: Hong(X,Y) — Hong(F(X),F(Y))
g \rightarrow F(g)

F FAITHFUL: F<sub>X,Y</sub> (NJ. 4X,Y)

F FULL: F<sub>X,Y</sub> SURJ. 4X,Y

F FULLY FAITHFUL: F<sub>X,Y</sub> BIJ. 4X,Y

F ESS. SURJ:

Y \( \in \) \( \frac{1}{2} \) \( \frac{1} \) \( \frac{1}{2} \) \( \frac{1}{2} \) \( \frac{1} \) \( \

6=9 
FINITER TIESTER THEOREM FUNCTOR F: 4 - 8 PF/(⇒) SAY ∃Ø: Ide = JAF \$ 4: FG => Ide AS IN CLAIM: FIS FULLY FAITHFUL AND ESS. SURJ. TAKE YEO. THEN FG(Y) => F ESS. SURJ. NOW tg:X -> X' & C GET X - 3 X' THIS IMPLIES FIS FAITHFAL SX = 2 = 1 8/21  $F(g) = F(\tilde{g}) \implies GF(g) = GF(\tilde{g})$   $\implies g = \tilde{g}$  $\begin{array}{c}
GF(X) \longrightarrow GF(X') \\
GF(3) & GF(3)
\end{array}$ 

CAND B

ARE EQUIVALENT IF

3F:C D & G:D D C

3F: Ide & FG = Ide

VRITE C = D

WRITE C = D

Fxiy: Hong(xiy) -> Hong(F(x), F(y))

g -> F(g)

F FAITHFUL: Fxiy INJ. YXiY

F FULL: Fxiy Surj. YXiY

F FULLY FAITHFUL: Fxiy BIJ. YXiY

F ESS. SURJ:

YYED, 3XEC ->. Y=F(x)

6=0 = FILLY FAITHFUL, ESS. SURJECTIVE THEOREM FUNCTOR F: 4 - 8 PF/(⇒) SAY ∃Ø: Ide = JAF \$ 4: FG => Ide AS IN CLAIM: FIS FULLY FAITHFUL AND ESS. SURJ. TAKE YED. THEN FG(Y) => Y => F ESS. SURJ. NOW tg: X -> X' e & GET X - 3 - X' THIS IMPLIES FIS FAITHFUL XX = 2 = 1 XX  $\begin{bmatrix}
F(g) = F(\tilde{g}) \Rightarrow GF(g) = GF(\tilde{g}) \\
\Rightarrow g = \tilde{g}
\end{bmatrix}
GF(x) \xrightarrow{GF(x')} GF(x')$ SWAPPING & WITH + => G IS FAITHFUL.

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& and & ARE EQUIVALENT IF 3F:6→0 \$ G:0→6 .<del>)</del>. GF= Ide & FG= Ide

 $f_{X,Y}$ :  $f_{X,Y}$ : F FAITHFUL: FXIY INJ. YXIY FFULL: Fxiy Surd. 4xiy FFALLY FAITHFAL: FXIY BIJ. YXIY F ESS. SURJ: YYED, 3xEG .. Y=F(x)

6=8 
FINITOR FILE THEOREM FUNCTOR F: 4 - B PF/(⇒) SAY ∃Ø: Ide = JAF \$ 4: FG => Ide AS IN CLAIM: FIS FULLY FAITHFUL AND ESS. SURJ. TAKE YEO. THEN FG(Y) => Y => F ESS. SURJ. NOW & g: X -> X' & C GET X - 3 - X' WRITE 6= B THIS IMPLIES FIS FAITHFAL &x = 2 = 1 8x  $\begin{bmatrix}
F(g) = F(\tilde{g}) \Rightarrow GF(g) = GF(\tilde{g}) \\
\Rightarrow g = \tilde{g}
\end{bmatrix}
GF(x) \xrightarrow{GF(x')} GF(x')$ SWAPPING & WITH + = G IS FAITHFUL. TAKE  $h: F(X) \rightarrow F(X') \in Q$ . BUILD 9: X => GF(X) G(L) GF(X') => X' & C.

CAND B

ARE EQUIVALENT IF

JF: C \rightarrow \$ \( \text{G:0} \rightarrow \)

JF: C \rightarrow \$ \( \text{G:0} \rightarrow \)

ARE EQUIVALENT IF

JF: C \rightarrow 0 \$ \( \text{FG} \rightarrow \)

ARE EQUIVALENT IF

JF: C \rightarrow 0 \$ \( \text{FG} \rightarrow \)

ARE EQUIVALENT IF

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ARE EQUIVALENT IF

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ARE EQUIVALENT IF

ARE EQUIVALENT IF

JF: C \rightarrow 0 \$ \( \text{FG} \rightarrow \)

ARE EQUIVALENT IF

ARE EQUIVALENT IF

JF: C \rightarrow 0 \$ \( \text{FG} \rightarrow \)

ARE EQUIVALENT IF

ARE EQUIVALENT

FXIY: HOME(X)Y) -> HOME(F(X), F(Y))

g -> F(g)

F FAITHFUL: FXIY INJ. YXIY

F FULL: FXIY SURJ. YXIY

F FULLY FAITHFUL: FXIY BIJ. YXIY

F ESS. SURJ:

YYED, JXEC. J. Y=F(X)

6=8 
FINITOR FILE THEOREM FUNCTOR F: 4 - 0 PF/(⇒) SAY ∃Ø: Ide = JAF & Y: FG => Ide AS IN NEED & DEFN CLAIM: FIS FULLY FAITHFUL AND ESS. SURJ. TAKE YEO. THEN FG(Y) => F ESS. SURJ. NOW & g: X -> X' & C, GET X - 3 - X' THIS IMPLIES FIS FAITHFUL SX = 2 = 1 8x'  $\begin{bmatrix}
F(g) = F(\tilde{g}) \Rightarrow GF(g) = GF(\tilde{g}) \\
\Rightarrow g = \tilde{g}
\end{bmatrix}
GF(x) \xrightarrow{GF(x')} GF(x')$ SWAPPING & WITH + = G IS FAITHFUL. TAKE  $h: F(X) \rightarrow F(X') \in Q$ . BUILD g: X = GF(X) GF(X') GF(X') X' & C.

THEN GF(g) = G(h). GFAITHFUL => F(g) = h

& FISFULL.

CAND B

ARE EQUIVALENT IF

JF: C-D & G: D-C

...

GF = Ide & FG = Ide

VALUENT IF

The FG

THE

Fry: Home (X,Y) -> Home (F(X), F(Y))

g -> F(g)

F FAITHFUL: Fx,y INJ. YX,Y

F FULL: Fx,y SURJ. YX,Y

FFULLY FAITHFUL: Fx,y BIJ. YX,Y

YYEB, 3xER 3. Y=F(x)

F ESS. SURJ:

6=0 = FINITED FILE THEOREM FUNCTOR F: 4 - 0 PF/(⇒) SAY ∃Ø: Ide = JAF & Y: FG => Ide AS IN NEED & DEFN CLAIM: FIS FULLY FAITHFUL AND ESS. SURJ. TAKE YEO. THEN FG(Y) => Y => F ESS. SURJ. NOW & g: X -> X' & C, GET X - 3 - X' THIS IMPLIES FIS FAITHFUL SX = 2 = 18x1  $\begin{aligned}
F(g) &= F(\tilde{g}) \implies GF(g) &= GF(\tilde{g}) \\
&\Rightarrow g &= \tilde{g}
\end{aligned}$   $GF(x) \longrightarrow GF(x')$  GF(g) = F(g) = GF(g) GF(g) = GF(g) GF(g) = GF(g) GF(g) = GF(g)SWAPPING & WITH + => G IS FAITHFUL. TAKE  $h: F(X) \rightarrow F(X') \in Q$ . BUILD 9: X => GF(X) G(L) GF(X') => X' & C. THEN GF(g) = G(h). GFAITHFUL => F(g) = h

& FIS FULL

CAND B

ARE EQUIVALENT IF

JF: C -> D & G: D -> C

...

GF == Ide & FG == Ide

CFF

Tide & Ide

WRITE C == D

PF/ (=) TAKE F: 4 - 0 FULLY FAITHFUL, ESS. SURJ.

FXIY: HOME(X,Y) -> HOME(F(X),F(Y))

g -> F(g)

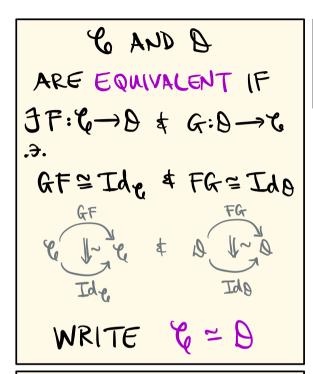
F FAITHFUL: FXIY INJ. YXIY

F FULL: FXIY SURJ. YXIY

F FULLY FAITHFUL: FXIY BIJ. YXIY

F ESS. SURJ:

YYED, 3XEC. 3. Y=F(X)



Frithful: Fxiy INJ. YXiY

FFULL: Fxiy SURJ. YXIY

FFULL: FXIY SURJ. YXIY

FFULL: FXIY SURJ. YXIY

FFULLY FAITHFUL: FXIY BIJ. YXIY

FESS. SURJ:

YYEB, JXEC. 3. Y=F(X)

CAND B

ARE EQUIVALENT IF

FF: C -> D & G: D -> C

...

GF == Ide & FG == Ide

CFF

Tide & Ide

WRITE C == D

Fx,y: Home(x,y) — Home(F(x),F(y))

g \rightarrow F(g)

F FAITHFUL: Fx,y (NJ. 4x,y)

F FULL: Fx,y SURJ. 4x,y

Y=F(x)

PF/ (=) TAKE F: 4 → 0 FULLY FAITHFUL, ESS. SURJ.

$$Y = (Y^5)^{\frac{1}{4}}$$
 .e.  $Y = Y \in Cans.223 = Y$ 

CAND B

ARE EQUIVALENT IF

JF: C -> D & G: D -> C

...

GF == Ide & FG == Ide

VRITE C == D

WRITE C == D

PF/ (=) TAKE F: 4 - 0 FULLY FAITHFUL, ESS. SURJ.

$$Y = (y5)7$$
 .e.  $Y = y56$   $Q = y4 \in CRU2.823 7$ 
 $Y = (y)8$ 
 $Y = (y)8$ 

- · ASSIGNMENTS Y & G(Y) MAKE A FUNCTOR G: B > C
  g & G(g)
- · 3 NATURAL ISOM Y: FG => Ido
- · 3 NATURAL ISOM Ø: Ide = GF

CAND B

ARE EQUIVALENT IF

FF: C -> D & G: D -> C

...

AF = Ide & FG = Ide

VRITE C = D

WRITE C = D

PF/ (=) TAKE F: 4 - 0 FULLY FAITHFUL, ESS. SURJ.

$$Y = (y5)7$$
 .e.  $Y = y5E$   $Q = y4 \in USUS.823 7$ 
 $Y = (y3)$ 
 $Y = (y3)$ 

- · ASSIGNMENTS Y HOGY) MAKE A FUNCTOR Q: 0-4 2 HOGY)

  TE FAITHFUL
- · 3 NATURAL ISOM Y: FG => Ido
- · 3 NATURAL ISOM Ø: Ide = GF

CAND B

ARE EQUIVALENT IF

FR: C -> D & G: D -> C

OF = Ide & FG = Ide

VRITE C - D

WRITE C - D

Fxiy: Home (xiy) -> Home (F(x), F(y))

g -> F(g)

F FAITHFUL: Fxiy INJ. Yxiy

F FULL: Fxiy Surd. Yxiy

Y = 555. Surd:

Y = 655. Surd:

PF/ (=) TAKE F: Y - O FULLY FAITHFUL, ESS. SURJ.

$$Y = (y5)7$$
 .e.  $Y = y56$   $Q = y4 \in CAU2.823 7$ 
 $Y = (y)3$ 
 $Y = (y)3$ 

- · ASSIGNMENTS Y HOGY) MAKE A FUNCTOR Q: 0-4 GHOGO TE FAITHFUL
- · 3 NATURAL ISOM Y: FG => Ido WITH COMPONENTS Y
- · J NATURAL ISOM Ø: Ide = GF

CAND B

ARE EQUIVALENT IF

JF: C \rightarrow D \cdot G: D \rightarrow C. D \rightarrow G: D \rightarrow C. D

Fxiy: Hong(X,Y) -> Hong(F(X),F(Y))
g -> F(g)

F FAITHFUL: Fxiy INJ. YX;Y

F FULL: Fxiy Surj. YX;Y

F FULLY FAITHFUL: Fxiy BIJ. YX;Y

F ESS. SURJ:

YYEB, 3xE& . Y=F(x)

THEOREM

G=0 

FINCTOR F: 4 - B

PF/ ( TAKE F: 4 - 0 FULLY FAITHFUL, ESS. SURJ.

$$Y = (y5)7$$
 e  $y = y56$   $0 = y4 \in Cans.823 7$ 
 $Y = (y)3$ 
 $Y = (y)3$ 

F FILLY FAITHFUL => Y g: Y -> Y' & B == 1! MORPHISM G(Y) -> G(Y') & & NEED TO SHOW

G(g)

- · ASSIGNMENTS Y H G(Y) MAKE A FUNCTOR G: B &

  SHORE)

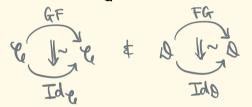
  TERAITHFUL
- · 3 NATURAL ISOM Y: FG => Ido WITH COMPONENTS Y
- · J NATURAL ISOM Ø: Ide = GF
  USING COMPONENTS YELD : F(X) -> FGF(X) & F FULLY
  FAITHFUL

DETAILS = EXER 234

# & dua &

ARE EQUIVALENT IF 3F:6-0 & G:0-6 ...

GF=Ide & FG=Ide



WRITE 6 = 0

 $F_{X,Y}$ : Home  $(X,Y) \rightarrow \text{thome}(F(X),F(Y))$   $g \mapsto F(g)$ 

F FAITHFUL: FXIY INJ. YXIY

FFULL: Fxiy Surd. 4xiy

FFACLY FAITHFAL: FXIY BIJ. YXIY

FESS. SURJ:

ΨY€B, 3×€& 3. Y=F(x)

# THEOREM G=0 FINCTOR F: 4 - 0

PF/ (=) TAKE F: 4 - 0 FULLY FAITHFUL, ESS. SURJ.

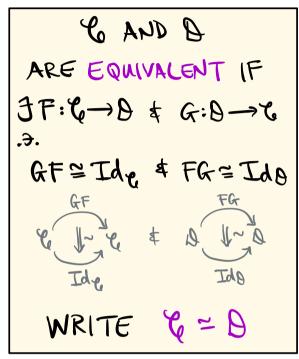
F FINLY FAITHFUL => Y g: Y -> Y' & B == 2! MORPHISM G(Y) -> G(Y') & C NEED TO SHOW

G(g)

- · ASSIGNMENTS Y H G(Y) MAKE A FUNCTOR G: B &

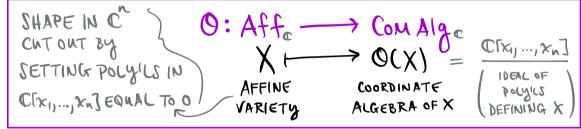
  SHORE)

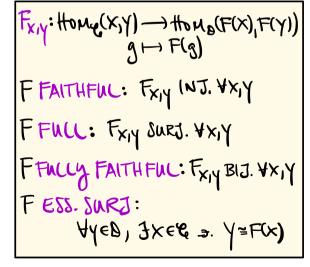
  TERAITHFUL
- · 3 NATURAL ISOM Y: FG => Ido WITH COMPONENTS Y
- · J NATURAL (SOM Ø: Ide => GF
  USING COMPONENTS Y=1 : F(X) -> FGF(X) & F FILLY
  FAITHFUL

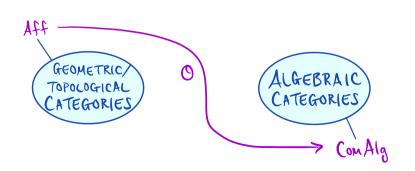


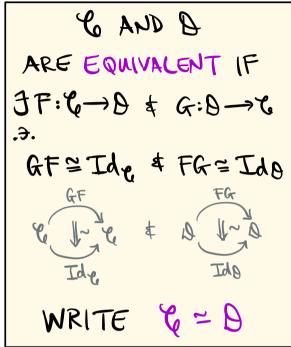


## TOWARD EXAMPLES—







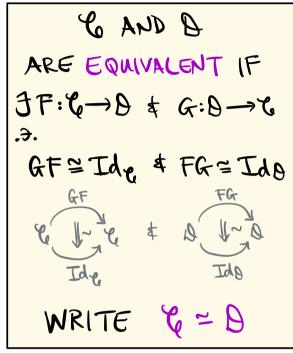


#### TOWARD EXAMPLES—

Ex. 
$$N=2$$

$$O\left(\frac{\sqrt{3}C^2}{\sqrt{3}}\right) = \frac{C(x_1y)}{(y)} \cong C(x)$$

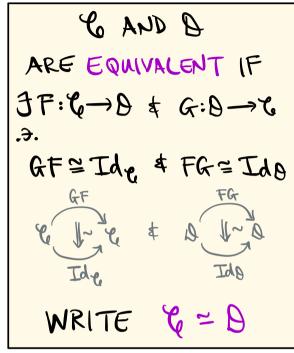
$$O\left(\frac{\sqrt{3}C^2}{\sqrt{3}}\right) = C(x_1y)$$

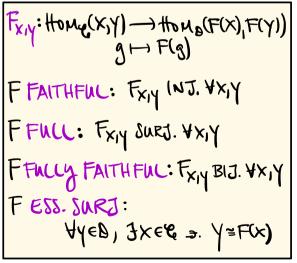


## TOWARD EXAMPLES -

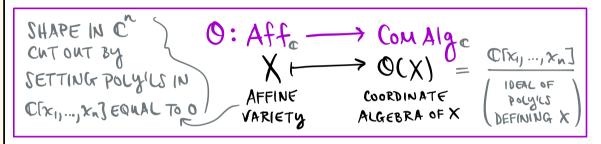
Ex. 
$$N=2$$

(NCCUSION  $O\left(\frac{3}{3}C^{2}\right) = \frac{C[x_{1}y]}{(y)} \cong C[x]$ 
 $O\left(\frac{3}{3}C^{2}\right) = C[x_{1}y]$ 



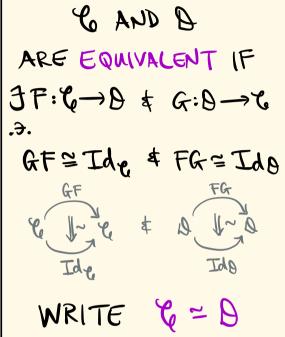


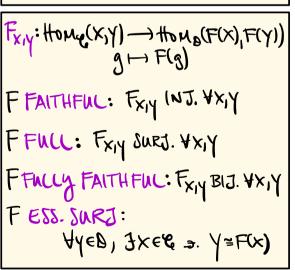
#### TOWARD EXAMPLES—

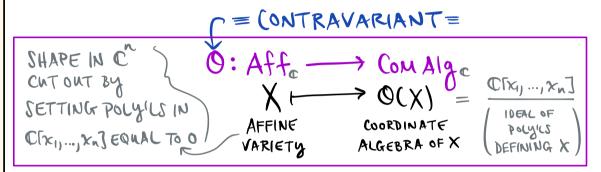


Ex. 
$$N=2$$
 $NCCUSION$ 
 $O(\frac{3}{7}C^2) = \frac{C[x_1y]}{77(y)} \cong C[x]$ 
 $O(\frac{3}{7}C^2) = C[x_1y]$ 

PROJECTION





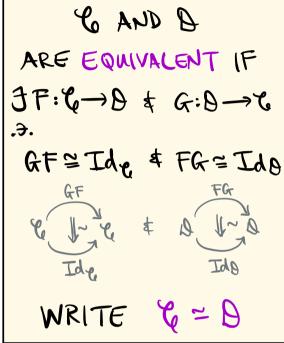


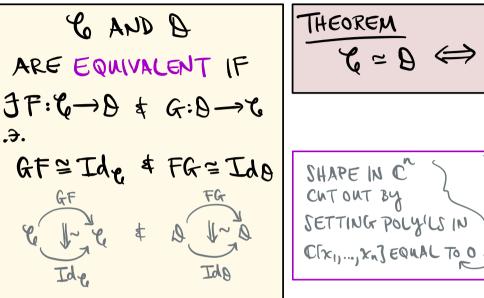
Ex. 
$$N=2$$

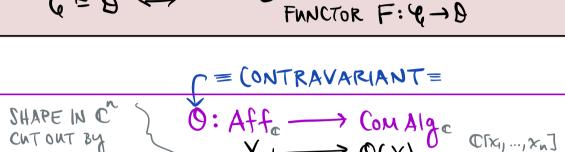
INCLUSION

 $O\left(\frac{3}{7}C^{2}\right) = \frac{C[x_{1}y]}{7} \cong C[x]$ 

PROJECTION







AFFINE

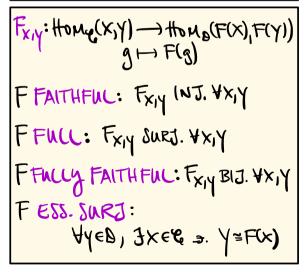
FRULY FAITHFUL, ESS. SURJECTIVE

COORDINATE

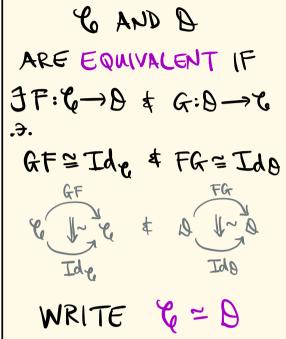
VARIETY ALGEBRA OF X

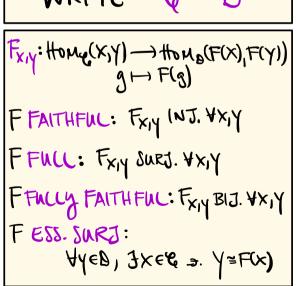
POLYICS

DEFINING X

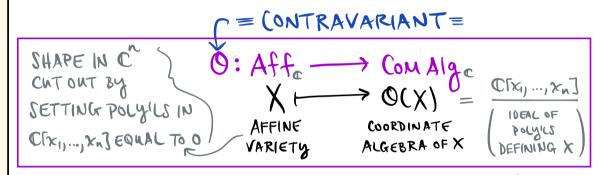


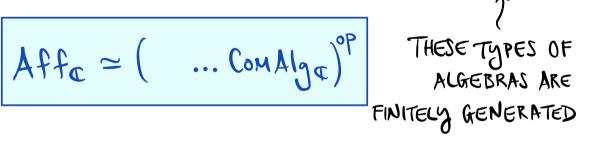
$$Aff_{\mathbb{C}} \simeq (\dots ComAlg_{\mathbb{C}})^{op}$$

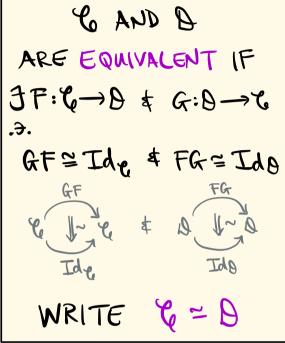


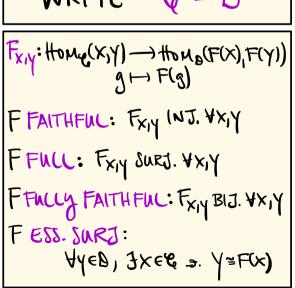


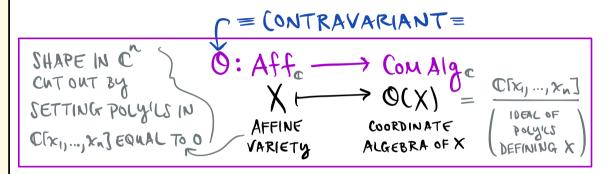




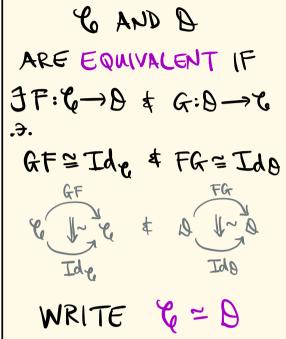


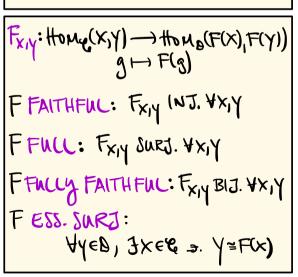


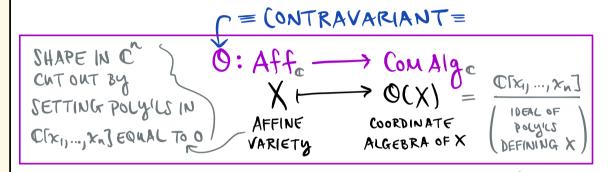












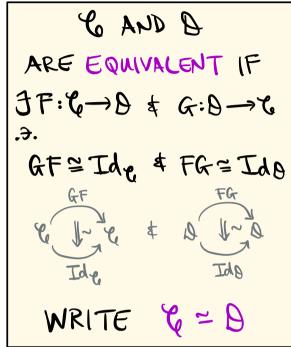
Affe 
$$\simeq$$
 (Fg ... ComAlge)°P

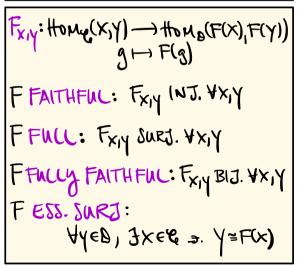
IN  $\mathbb{C}^2_{(x,y)}$ :

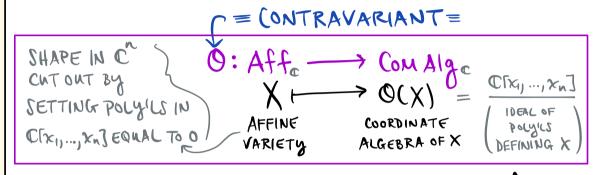
THE SHAPE 
$$\chi = y = 0$$
  
(ORIGIN)

THE SHAPE  $\chi^2 = y = 0$   
(ORIGIN)

THESE TYPES OF ALGEBRAS ARE FINITELY GENERATED







Affe 
$$\simeq$$
 (Fg ... ComAlg  $\varphi$ ) These types of algebras are finitely generated

The shape  $\chi = y = 0$ 

The shape  $\chi^2 = y = 0$ 

Corigin)

The shape  $\chi^2 = y = 0$ 

Corigin)

Corigin

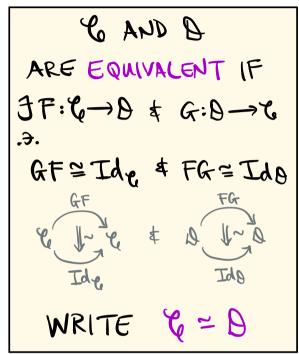
Corigin

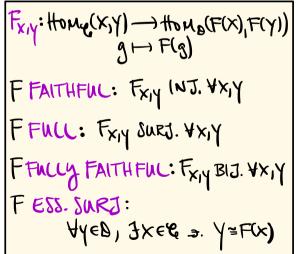
Corigin

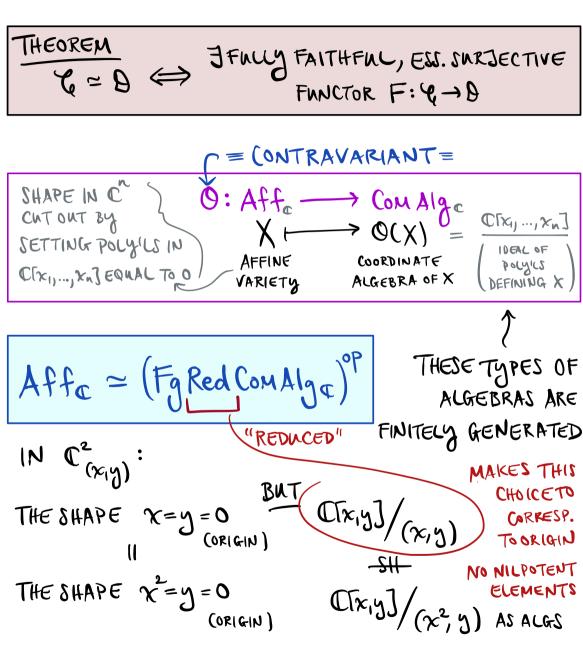
Corigin

Corigin

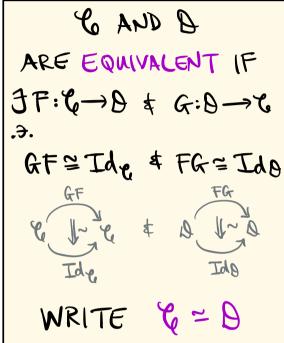
THESE TYPES OF

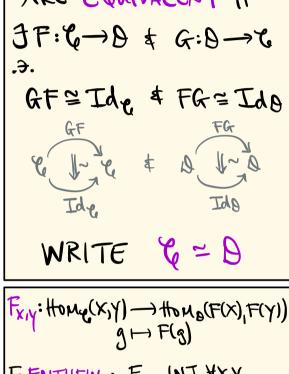


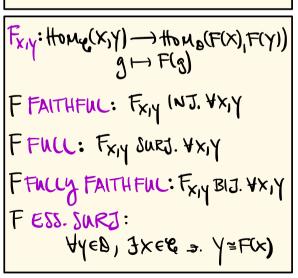




(ORIGIN)







SHAPE IN 
$$C^{n}$$

CHONTRAVARIANT =

SHAPE IN  $C^{n}$ 

CHOOK BY

SETTING POLYILS IN

AFFINE COORDINATE

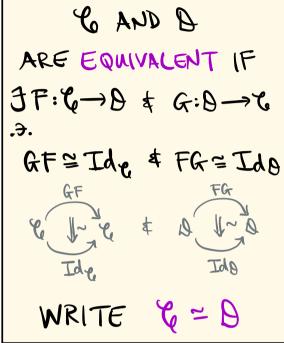
POLYILS

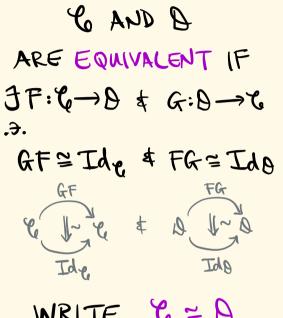
VARIETY

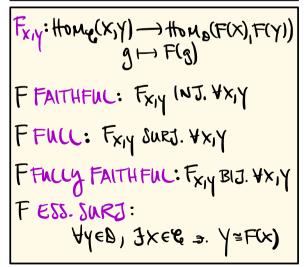
ALGEBRA OF X

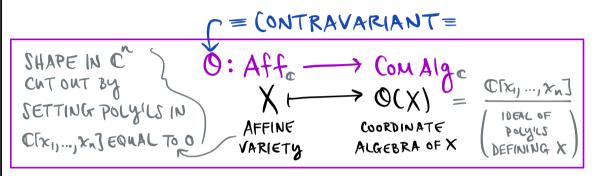
DEFINING X

ALSO HAVE



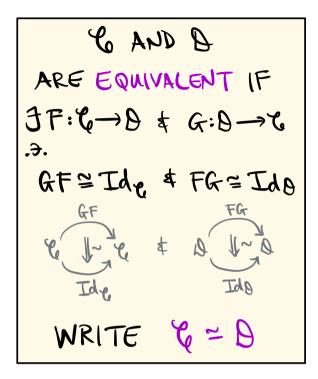






ALSO HAVE

Scheme = (ComAlge)°P



# NOTION OF SAMENESS FOR IK-ALGEBRAS

CAND B

ARE EQUIVALENT IF

JF: C -> D & G: D -> C

...

GF == Ide & FG == Ide

GF

Tde

WRITE C == D

# NOTION OF SAMENESS FOR IK-ALGEBRAS

A = B EQUALITY OF ALGEBRAS

CAND B

ARE EQUIVALENT IF

JF: C -> D & G: D -> C

...

GF = Ide & FG = Ide

GF

Tide

WRITE & = D

NOTION OF SAMENESS FOR 1k-ALGEBRAS

A = B EQUALITY OF ALGEBRAS

WEAKEN > ISOMORPHISM
OF ALGEBRAS

CAND B

ARE EQUIVALENT IF

JF: C -> D & G: D -> C

SF == Ido

FG == Ido

WRITE C -= D

WRITE C -= D

TAKE IK-AUGS A & B

A IS MORITA EQUIV. TO B IF A-Mod = B-Mod.

THAT IS, A & B HAVE THE SAME REP THY

## NOTION OF SAMENESS FOR IK-ALGEBRAS

A = B EQUALITY OF ALGEBRAS

WEAKEN > ISOMORPHISM
OF ALGEBRAS

WEAKEN > A MORITA B
"EQUIVALENCE"
OF ALGEBRAS

CAND B

ARE EQUIVALENT IF

JF: C -> D & G: D -> C

...

GF = Ide & FG = Ide

GF

Tde

WRITE C = D

TAKE IK-ALGS A & B

A IS MORITA EQUIV. TO B

IF A-Mod = B-Mod.

THAT IS, A & B HAVE

THE SAME REP THY

CAND B

ARE EQUIVALENT IF

3F: C -> D & G: D -> C

3.

GF == Ide & FG == Ide

VRITE C == D

WRITE C == D

MORITA'S THEOREM A IS MORITA EQUIVALENT TO B

A B MODULES APB & BQA . 7.

POBQ = Areg AS A-BIMODULES

Q Q P = Brey AS B-BIMODULES.

TAKE  $F := Q \otimes_A -: A - Mod \longrightarrow B - Mod$   $G := P \otimes_B -: B - Mod \longrightarrow A - Mod$ 

TAKE IK-ALGS A & B

A IS MORITA EQUIV. TO B

IF A-Mod = B-Mod.

THAT IS, A & B HAVE

THE SAME REP THY

CAND B

ARE EQUIVALENT IF

JF: C -> D & G: D -> C

...

GF == Ide & FG == Ide

VRITE & == D

TAKE IK-ALGS A & B

A IS MORITA EQUIV. TO B IF A-Mod = B-Mod.

THAT IS, A & B HAVE THE SAME REP THY MORITA'S THEOREM A IS MORITA EQUIVALENT TO B

A BIMODULES APB & BQA . 7.

POBQ = Areg AS A-BIMODULES

Q Q P = Brey AS B-BIMODULES.

 $PF/(\rightleftharpoons)$ TAKE  $F:=Q\otimes_{A}-:A-M\circ d\longrightarrow B-M\circ d$   $G:=P\otimes_{B}-:B-M\circ d\longrightarrow A-M\circ d$ 

NOW Y MEA-mod, GET:

$$GF(M) = G(Q_{\varnothing_A}M) = P_{\varnothing_B}(Q_{\varnothing_A}M)$$

$$\stackrel{=}{=} (P_{\varnothing_B}Q)_{\varnothing_A}M$$

Modify EXER.1.186

CAND B

ARE EQUIVALENT IF

FR: C-D & G: B-> C

AF = Ide & FG = Ide

CF = Ide

WRITE C = B

TAKE IK-ALGS A & B

A IS MORITA EQUIV. TO B IF A-Mod = B-Mod.

THAT IS, A & B HAVE THE SAME REP THY  $PF/\iff$ TAKE  $F:=Q\otimes_{A}-:A-M\circ d\longrightarrow B-M\circ d$   $G:=P\otimes_{B}-:B-M\circ d\longrightarrow A-M\circ d$ 

NOW Y MEA-mod, GET:

GF(M) = 
$$G(Q_{\otimes_A}M) = P_{\otimes_B}(Q_{\otimes_A}M)$$
  
=  $(P_{\otimes_B}Q)_{\otimes_A}M = A_{eg}\otimes_A M = M$ .  
EXER.1.18a

CAND B

ARE EQUIVALENT IF

FR: C-D & G: B-C

...

GF = Ide & FG = Ide

CFF & Ide & FG = Ide

The Ide Ide Ide

The Ide Ide

WRITE 6 = 0

TAKE IK-ALGS A \$ B

A IS MORITA EQUIV. TO B IF A-Mod = B-Mod.

THAT IS, A & B HAVE THE SAME REP THY MORITA'S THEOREM A IS MORITA EQUIVALENT TO B

€ 3 BIMODULES APB & BQA .7.

POBQ = Areg AS A-BIMODULES \$\Display Q \otimes\_AP = Breg AS B-BIMODULES.

PF/ (<del>←</del>)

TAKE F := Q & -: A-Mod -> B-Mod

 $G := P \otimes_R - : B - M \circ d \longrightarrow A - M \circ d$ 

NOW Y MEA-MOD, GET:

 $GF(M) = G(Q_{\otimes_{A}}M) = P_{\otimes_{B}}(Q_{\otimes_{A}}M)$ =  $(P_{\otimes_{B}}Q)_{A}M = A_{Reg}\otimes_{A}M = M$ .

: GF = Id\_-mod.

LIKEWISE, FG = Id B-mod.

CAND B

ARE EQUIVALENT IF

JF: C - O & G: O -> C

...

GF = Ide & FG = Ide

VRITE C = O

WRITE C = O

 $PF/(\Longrightarrow)$  GIVEN EQUIVALENCE  $F:A-Mod \to B-Mod$ TAKE  $Q:=F(_AAreg) \in B-Mod$ 

TAKE IK-ALGS A & B

A IS MORITA EQUIV. TO B

IF A-Mod = B-Mod.

THAT IS, A & B HAVE

THE SAME REP THY

CAND B

ARE EQUIVALENT IF

F: C -> B & G: B -> C

...

AF = Ide & FG = Ide

VILLE & Ide

WRITE & = B

WRITE & = B

TAKE IK-AUGS A & B

A IS MORITA EQUIV. TO B

IF A-Mod = B-Mod.

THAT IS, A & B HAVE

THE SAME REP THY

PF/ (=>) GIVEN EQUIVALENCE F: A-Mod -> B-Mod

TAKE Q := F(A Areg) & B-Mod

GET AOP = End\_A-mod (A Areg)

= End\_B-mod (F(A Areg))

= End\_B-mod (BQ)

CAND B

ARE EQUIVALENT IF

FIGORIA GIBOR

FIGURALENT IF

GIBOR

FIGURALENT IF

FI

TAKE IK-ALGS A & B

A IS MORITA EQUIV. TO B

IF A-Mod = B-Mod.

THAT IS, A & B HAVE

THE SAME REP THY

PF/ (=>) GIVEN EQUIVALENCE F: A-Mod -> B-Mod

TAKE Q := F(A Areg) & B-Mod

GET AOP = End\_A-mod (A Areg)

EXER. 1.26 = End\_B-mod (F(A Areg))

FRILLY = End\_B-mod (BQ)

CAND B

ARE EQUIVALENT IF

FR: C -> D & G: D -> C

OF = Ide & FG = Ide

VRITE C -= D

WRITE C -= D

TAKE IK-ALGS A & B

A IS MORITA EQUIV. TO B

IF A-Mod = B-Mod.

THAT IS, A & B HAVE THE SAME REP THY MORITA'S THEOREM A IS MORITA EQUIVALENT TO B

A BIMODULES APB & BQA . 7.

POBQ = Areg AS A-BIMODULES

QOAP = Brey AS B-BIMODULES.

PF/ (=>) GIVEN EQUIVALENCE F: A-Mod -> B-Mod

TAKE Q := F(A Areg) & B-Mod

GET A<sup>op</sup> = End<sub>A-mod</sub> (A Areg)

= End<sub>B-mod</sub> (F(A Areg))

= End<sub>B-mod</sub> (BQ)

DEFINE 9 4 A := f(a)(q)

~> BQ ∈ (B,A)-Bimod

CAND B ARE EQUIVALENT IF  $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \ddagger G: 0 \rightarrow C$   $JF: C \rightarrow 0 \Rightarrow C$  $JF: C \rightarrow 0 \Rightarrow C$ 

MORITA'S THEOREM A IS MORITA EQUIVALENT TO B

A B MODULES APB & BQA . 7.

POBQ = Areg AS A-BIMODULES

Q Q P = Brey AS B-BIMODULES.

 $PF/(\Longrightarrow)$  GIVEN EQUIVALENCE  $F:A-Mod \to B-Mod$ HAVE  $Q:=F(_AAreg)\in (B,A)-Bimod$ 

TAKE IK-ALGS A & B

A IS MORITA EQUIV. TO B

IF A-Mod = B-Mod.

THAT IS, A & B HAVE

THE SAME REP THY

CAND B

ARE EQUIVALENT IF

F: C -> B & G: B -> C

...

GF == Ide & FG == Ide

CFF

Tide

WRITE C == B

WRITE C == B

TAKE IK-ALGS A & B

A IS MORITA EQUIV. TO B

IF A-Mod = B-Mod.

THAT IS, A & B HAVE

THE SAME REP THY

MORITA'S THEOREM A IS MORITA EQUIVALENT TO B

A B MODULES APB & BQA . 7.

POBQ = Areg AS A-BIMODULES

Q Q P = Brey AS B-BIMODULES.

 $PF/(\Longrightarrow)$  GIVEN EQUIVALENCE  $F:A-Mod \to B-Mod$ HAVE  $Q := F(_A Areg) \in (B,A)-Bimod$   $CLAIM : F \cong Q \otimes_A - AS FUNCTORS$ 

CAND B

ARE EQUIVALENT IF

JF: C -> B & G: B -> C

...

GF = Ide & FG = Ide

VRITE C = B

WRITE C = B

TAKE IK-AUGS A & B

A US MORITA EQUIV. TO B

IF A-Mod = B-Mod.

THAT IS, A & B HAVE

THE SAME REP THY

PF/  $\iff$  GIVEN EQUIVALENCE F: A-Mod  $\rightarrow$  B-Mod

HAVE Q:= F(\_A Areg)  $\in$  (B, A)-Bimod

CLAIM: F= Q  $\otimes_A$  — As FUNCTORS

PF/ TAKE ME A-Mod & GET (80:  $\sigma_X: X = \text{Hom}_{A-\text{mod}}(A,X) \xrightarrow{F} \text{Hom}_{B-\text{mod}}(F(A),F(X))$ FILLY HOMB-Mod (Q, F(X))

TAITHFUL

CAND B

ARE EQUIVALENT IF

JF: C - O & G: O -> C

...

GF = Ide & FG = Ide

VRITE C = O

WRITE C = O

TAKE IK-AUGS A & B

A US MORITA EQUIV. TO B

IF A-Mod = B-Mod.

THAT IS, A & B HAVE

THE SAME REP THY

PF/  $(\Longrightarrow)$  GIVEN EQUIVALENCE F: A-Mod  $\to$  B-Mod

HAVE  $Q := F(_A Areg) \in (B_1A) - Bimod$ CLAIM:  $F \cong Q \otimes_A - As$  Functors

PF/ Take Me A-Mod & GET (So:  $\sigma_X : X \cong Hom_{A-mod}(A_1X) \xrightarrow{F} Hom_{B-mod}(F(A), F(X))$ TENSOR-Hom ADJUNCTION Hom\_B-mod (Q, F(X))

Hom\_B-mod (Q \( \text{Q} \text{X}, \( \text{Y} \))  $\cong Hom_{A-mod}(X, Hom_{B-mod}(Q, Y))$ GET (So:  $\sigma_X : Q \otimes_A X \longrightarrow F(X)$ 

CAND B

ARE EQUIVALENT IF

F: C -> B & G: B -> C

...

AF = Ide & FG = Ide

VRITE C = B

WRITE C = B

TAKE IK-ALGS A & B

A IS MORITA EQUIV. TO B

IF A-Mod = B-Mod.

THAT IS, A & B HAVE

THE SAME REP THY

MORITA'S THEOREM A IS MORITA EQUIVALENT TO B

A B MODULES APB & BQA . 7.

POBQ = Areg AS A-BIMODULES

Q Q P = Breg AS B-BIMODULES.

PF/ (⇒) GIVEN EQUIVALENCE F: A-Mod → B-Mod HAVE Q := F(A Area) E(B,A)-Bimod CLAIM: F= Q ØB - AS FUNCTOR PF/ TAKE ME A-Mod & GET 180:  $\sigma_{X}: X = Hom_{A-mod}(A,X) \xrightarrow{F} Hom_{B-mod}(F(A),F(X))$ TENSOR-HOM ADJUNCTION HOMB-MOD (Q, FCXI) HomB-mod (Q & X, Y) = HomA-mod (X, HomB-mod (Q, Y)) GET (So:  $\sigma_{X}': Q \otimes_{A} X \longrightarrow F(X) \longrightarrow Q \otimes_{A} - \stackrel{\bullet}{\Longrightarrow} F$ 

 $PF/(\Longrightarrow)$  GIVEN EQUIVALENCE  $F: A-Mod \to B-Mod$ HAVE  $Q := F(_A Areg) \in (B_1A)-Bimod$ HAVE  $F \cong Q \otimes_A - AS FUNCTORS$ 

TAKE IK-AUGS A & B

A IS MORITA EQUIV. TO B

IF A-Mod = B-Mod.

THAT IS, A & B HAVE

THE SAME REP THY

CAND B

ARE EQUIVALENT IF

F: C -> B & G: B -> C

...

AF = Ide & FG = Ide

VILLE & Ide

WRITE & = B

WRITE & = B

TAKE IK-ALGS A & B

A IS MORITA EQUIV. TO B

IF A-Mod = B-Mod.

THAT IS, A & B HAVE

THE SAME REP THY

MORITA'S THEOREM A IS MORITA EQUIVALENT TO B

A BIMODULES APB & BQA . 7.

POBQ = Areg AS A-BIMODULES

Q Q P = Brey AS B-BIMODULES.

PF/ $(\Longrightarrow)$  GIVEN EQUIVALENCE F: A-Mod  $\to$  B-Mod HAVE Q:= F(A Areg)  $\in$  (B, A) - Bimod HAVE F  $\cong$  Q  $\otimes_A$  - AS FUNCTORS NOW  $\exists$  G: B-Mod  $\to$  A-Mod with

Ø: IdAmod => GF & Y: FG => IdB-Mod

TAKE IK-AUGS A & B

A IS MORITA EQUIV. TO B

IF A-Mod = B-Mod.

THAT IS, A & B HAVE

THE SAME REP THY

PF/ ( $\Rightarrow$ ) GIVEN EQUIVALENCE F: A-Mod  $\rightarrow$  B-Mod HAVE Q:= F(\_A Areg)  $\in$  B, A) - Bimod HAVE F  $\cong$  Q  $\otimes_A$  - AS FUNCTORS NOW  $\exists$  G: B-Mod  $\rightarrow$  A-Mod with  $\not S: \text{Id}_{A-\text{mod}} \xrightarrow{\cong} \text{GF} \quad \Leftrightarrow \quad \text{Y:FG} \xrightarrow{\cong} \text{Id}_{B-\text{mod}}$ HAVE  $F \cong F \otimes_B - \text{AS FUNCTORS}$ 

CAND B

ARE EQUIVALENT IF

F: C - O & G: O -> C

...

AF = Ide & FG = Ide

VITE & Ide

WRITE & = O

WRITE & = O

WRITE & = O

TAKE IK-ALGS A & B

A IS MORITA EQUIV. TO B

IF A-Mod = B-Mod.

THAT IS, A & B HAVE

THE SAME REP THY

MORITA'S THEOREM A IS MORITA EQUIVALENT TO B

A BIMODULES APB & BQA . 7.

POBQ = Areg AS A-BIMODULES

QOAP = Brey AS B-BIMODULES.

PF/ (⇒) GIVEN EQUIVALENCE F: A-Mod → B-Mod

HAVE  $Q := F(_A Areg) \in (B,A) - Bimod$ HAVE  $F \cong Q \otimes_A - AS$  FUNCTORS

NOW  $\exists G : B - Mod \rightarrow A - Mod \text{ with}$   $\varnothing : Td_{A-mod} \cong GF \Leftrightarrow \Upsilon : FG \cong Td_{B-mod}$ HAVE  $P := G(_B Breg) \in (A,B) - Bimod$ HAVE  $G \cong P \otimes_B - AS$  FUNCTORS

GET  $\varphi_A : A \cong GF(A) \cong P \otimes_B Q$  As A - Bimods  $\Leftrightarrow \Psi_B^{-1} : B \cong FG(B) \cong Q \otimes_A P$  As B - Bimods

DETAILS = EXER. 2.35

& and &

ARE EQUIVALENT IF  $3F:C \rightarrow 0 + G:0 \rightarrow C$ .

GF= Ide & FG= Ide



WRITE &= B

TAKE IK-AUGS A & B

A IS MORITA EQUIV. TO B IF A-Mod = B-Mod.

THAT IS, A & B HAVE THE SAME REP THY  $PF/(\Longrightarrow)$  GIVEN EQUIVALENCE  $F: A-Mod \to B-Mod$ HAVE  $Q := F(_A Areg) \in (B,A)-Bimod$ HAVE  $F \cong Q \otimes_A - AS$  FUNCTOR

CAND B

ARE EQUIVALENT IF

JF: C-D & G: D-C

ARE EQUIVALENT IF

JF: C-D & FG = Ido

FG = Ido

WRITE

WRITE

MAIN EXAMPLE

A 13 MORITA EQUIVALENT TO Matn(A)

TAKE IK-AUGS A & B

A IS MORITA EQUIV. TO B IF A-Mod ~ B-Mod.

THAT IS, A & B HAVE THE SAME REP THY

CAND B

ARE EQUIVALENT IF

JF: C -> D & G: D -> C

...

GF == Ide & FG == Ide

VRITE C == D

WRITE C == D

TAKE IK-AUGS A & B

A IS MORITA EQUIV. TO B IF A-Mod = B-Mod.

THAT IS, A & B HAVE THE SAME REP THY MAIN EXAMPLE

A IS MORITA EQUIVALENT TO Matn(A)

VIA  $P = \{(a_1, ..., a_n) \mid a_i \in A\} \in (A, Matn(A)) - Binod$ 

$$Q = \left\{ \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \middle| a_i \in A \right\} \in (Mat_n(A), A) - Bimod$$

CAND B

ARE EQUIVALENT IF

JF: C -> D & G: D -> C

...

GF == Ide & FG == Ide

VRITE C == D

WRITE C == D

TAKE IK-ALGS A & B

A IS MORITA EQUIV. TO B

IF A-Mod = B-Mod.

THAT IS, A & B HAVE

THE SAME REP THY

MAIN EXAMPLE

A 1S MORITA EQUIVALENT TO Matn(A)

VIA  $P = \{(a_1, ..., a_n) \mid a_i \in A \} \in (A, Matn(A)) - Bimod$   $a' b (a_1, ..., a_n) := (a'a_1, ..., a'a_n)$   $(a_1, ..., a_n) \triangleleft A' := \underline{a} A' \quad \text{MATRIX MULTIP'N}$   $Q = \begin{cases} (a_1) \mid a_i \in A \\ \vdots \mid a_n \end{cases} \in (Matn(A), A) - Bimod$  LIKEWISE

CAND B

ARE EQUIVALENT IF

JF: C -> B & G: B -> C

...

GF == Ide & FG == Ide

VALUE & FG == Ide

WRITE & == B

MORITA'S THEOREM A IS MORITA EQUIVALENT TO B

A B MODULES APB & BQA . 7.

POBQ = Areg AS A-BIMODULES

Rep AS B-BIMODULES.

MAIN EXAMPLE

A 13 MORITA EQUIVALENT TO Matn(A)

TAKE IK-AUGS A & B

A IS MORITA EQUIV. TO B

IF A-Mod = B-Mod.

THAT IS, A & B HAVE THE SAME REP THY

CAND B

ARE EQUIVALENT IF

FF: C -> B & G: B -> C

...

AF = Ide & FG = Ide

CFF & DITTE

WRITE & = B

MORITA'S THEOREM A IS MORITA EQUIVALENT TO B

A BIMODULES APB & BQA . 7.

POBQ = Areg AS A-BIMODULES

Rep AS B-BIMODULES.

MAIN EXAMPLE

A 13 MORITA EQUIVALENT TO Matn(A)

TAKE IK-AUGS A & B

A IS MORITA EQUIV. TO B

IF A-Mod = B-Mod.

THAT IS, A & B HAVE

THE SAME REP THY

A IS MORITA EQUIV. TO B

IF A-Mod = B-Mod.

EXER.2.38 A~mor B => 72(A) = 2(B)

CENTER OF A

& and & ARE EQUIVALENT IF 3F:6→B \$ G:B→6 .<del>)</del>. GF=Ide & FG=Ide 1-je + 0 1-jo WRITE 6 = 0

TAKE IK-ALGS A & B

IF A-Mod = B-Mod.

THAT IS, A & B HAVE THE SAME REP THY

MORITA'S THEOREM A IS MORITA EQUIVALENT TO B € 3 BIMODULES APB & BQA .7. POBQ = Area AS A-BIMODULES \$ Q ØA P = Brey AS B-BIMODULES.

MAIN EXAMPLE

A IS MORITA EQUIVALENT TO Matn(A)

FOR INSTANCE:

Ik ~mor Matn(k) & Z(Matn(k)) = Ik

A IS MORITA EQUIV. TO B EXER. 2.38 A ~ MOR B => 72(A) = 2(B)

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CAND B

ARE EQUIVALENT IF

JF: C -> D & G: D -> C

...

GF == Ide & FG == Ide

VRITE C == D

WRITE C == D

TAKE IK-ALGS A & B

A IS MORITA EQUIV. TO B

IF A-Mod = B-Mod.

THAT IS, A & B HAVE

THE SAME REP THY

EXER.2.38 A ~ MOR B REPART CENTER OF A

FOR COMM.

ALGS C, C'

C MOR C'

ALGS C, C'

C MOR C'

ALGS C, C'

C MOR C'

REVERAL

CENTER OF A

2(C) = 2(C')

## = SUMMARY =

NOTION OF SAMENESS FOR IK-ALGEBRAS A = BEQUALITY OF ALGEBRAS "EQUIVALENCE" OF ALGEBRAS

## = SUMMARY =

NOTION OF SAMENESS NOTION OF SAMENESS FOR CATEGORIES FOR IK-ALGEBRAS C = 0A = BEQUALITY EQUALITY OF ALGEBRAS OF CATEGORIES OF CATEGORIES "EQUIVALENCE" EQUIVALENCE OF ALGEBRAS OF CATEGORIES

## = SUMMARY =

NOTION OF SAMENESS NOTION OF SAMENESS FOR CATEGORIES FOR IK-ALGEBRAS C = DA = BEQUALITY EQUALITY OF ALGEBRAS OF CATEGORIES OF CATEGORIES "EQUIVALENCE" EQUIVALENCE OF ALGEBRAS OF CATEGORIES NEXT TIME: ADJUNCTION

MATH 466/566 SPRING 2024

CHELSEA WALTON RICE U.

LECTURE #9

## TOPICS:

I. ISOMORPHISM OF CATEGORIES (§2.4.1)

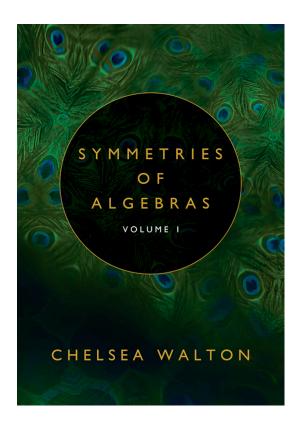
II. EQUIVALENCE OF CATEGORIES (552.4.2-2.4.3)

III. MORITA EQUIVALENCE (§2.4.3)

NEXT TIME: ADJUNCTION

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<u>Lecture #9 keywords</u>: equivalence of categories, isomorphism of categories, Morita equivalence of algebras, Morita's Theorem, skeleton