Abstract. The Hopf-Laplace equation arises in the study of the Dirichlet energy integral

$$E[h] = \iint_X |Dh|^2 = 2 \iint_X (|h_z|^2 + |h_{\bar{z}}|^2) \,dz$$

for mappings $h: \mathbb{X} \to \mathbb{Y}$ between two designated domains $\mathbb{X}$ and $\mathbb{Y}$ in the complex plane. Here and throughout we take advantage of the complex partial derivatives, a notation indispensable for advancing this work. The inner variation of the energy leads to a second order nonlinear system of PDEs for a complex function $h$ in the Sobolev space $W^{1,2}(\mathbb{X}, \mathbb{Y})$,

(\ast) \quad \frac{\partial}{\partial \bar{z}} (h_z \overline{h_{\bar{z}}}) = 0, \quad \text{in the sense of distributions.}

This is what we call the Hopf-Laplace equation in its natural domain of definition. The classical solutions are the complex harmonic maps - the solutions of the linear Laplace system $h_z = 0$. It is true, though not obvious at all, that homeomorphic $W^{1,2}$-solutions of (\ast) are indeed harmonic diffeomorphisms. Nevertheless, minimization of the Dirichlet energy among homeomorphisms often leads to noninjective extremal mappings, thus nonharmonic solutions of (\ast) (so-called squeezing phenomenon). We investigate the equation (\ast) for a certain class of topologically well behaved mappings which are almost homeomorphisms, called Hopf deformations. The associated Hopf quadratic differential $(h_z \overline{h_{\bar{z}}}) \,dz^2$ and its trajectories enter the stage. We have established Lipschitz continuity of Hopf deformations, the best possible regularity one can get, since in general Hopf deformations are not $C^1$-smooth. Thus in particular, we show that the minimal-energy deformations are Lipschitz continuous, a result of considerable interest in the theory of minimal surfaces, calculus of variations and PDEs, with potential applications to elastic plates.

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