Singularities of special Lagrangian submanifolds and SYZ

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Special Lagrangian m-folds (SL m-folds) are m-dimensional calibrated submanifolds in Calabi–Yau m-folds (which have real dimension $2m$). They are important in String Theory (as the classical limits of 'A-branes'), and are key ingredients of the SYZ Conjecture, a geometric explanation of Mirror Symmetry. One can also argue that SL 3-folds are the simplest of the nontrivial calibrated geometries associated to special holonomy groups (excluding complex submanifolds, which are well understood) so we should try to understand them first, before going on to the higher geometries.

This talk will survey my research on special Lagrangian singularities, which focusses on compact singular SL m-folds in (almost) Calabi-Yau m-folds. General singularities of SL m-folds can be very bad and difficult to study, so it makes sense to look for a class of 'well-behaved' singular SL m-folds with better properties. Such a class are SL m-folds with *isolated conical singularities*, which have finitely many singular points modelled on the vertex of an SL cone in $\mathbf{C}^m$ singular only at 0. These have a well-behaved deformation theory with finite-dimensional obstruction spaces, are quite stable under deformations of the underlying (almost) Calabi–Yau m-fold, and can be *desingularized* by gluing in *Asymptotically Conical* SL m-folds in at the singular points.

Then we discuss the *SYZ Conjecture*, which explains Mirror Symmetry of Calabi–Yau 3-folds $M, \hat{M}$ in terms of *special Lagrangian fibrations* $f : M \to B$, $\hat{f} : \hat{M} \to B$ over the same base B. We are interested in what can be said about the singularities of $f, \hat{f}$, particularly when $M, \hat{M}$ are *generic*. We construct U(1)-invariant local models of fibrations in $\mathbf{C}^3$, and argue that $f, \hat{f}$ are not smooth, and their singularities are of codimension 1 in B.

Some references for this talk are math.DG/0310460, math.DG/0206016 and math.DG/0011179.