

Singularities and blow up for solutions of the Yamabe equation

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Abstract: Many nonlinear geometric problems have solutions which exhibit singularities, and sequences of solutions which blow up due to a failure of estimates. For Yamabe-type equations both of these phenomena occur in certain cases, and can be ruled out in others depending on the geometric situation. For singular solutions it is natural to attempt to classify the possible singular sets which can occur. We will describe recent results of Denis Labutin which characterize singular sets (sets at infinity) for Yamabe solutions of negative or zero scalar curvature in terms of appropriate capacities. We then describe recent joint work with Marcus Khuri which shows that local solutions of the Yamabe equation for positive scalar curvature can only blow up at points at which the Weyl curvature of the background metric vanishes to a precise order. We briefly describe how this result completes an old program for obtaining global estimates for Yamabe solutions on arbitrary compact manifolds.