

Undergraduate Mathematics Colloquium

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“Energy-Minimizing Unit Vector Fields in Three Space”

Tuesday September 3

Talk at 4:00 in Herman Brown 227

Tea & cookies at 3:30 in Herman Brown 438

Pizza after the talk

ABSTRACT: Given a smooth real-valued function g on the unit sphere S in \mathbb{R}^3 , the function u on the closed unit ball B which coincides with g on S and has least energy $\int_B |Du|^2 dx$ is a *harmonic function*. These show up in classical mathematics, physics, and many applications; e.g. u represents the longtime steady-state temperature distribution resulting from keeping the surface temperature equal to g .

Later, many other interesting problems required changing real-valued functions to *unit* vector fields. This seemingly small change led to many new problems and results. We will look at some of the basic examples, questions and solutions including:

1. Is there always a minimizer? Is it always unique? Always continuous? (yes,no,no)
2. What is the minimizing unit vector field u on B such that $u(x) = x$ for x in S ? ($x/|x|$)
3. Can minimizers have isolated singularities of any degree? (no)
4. Can minimizers have 1 dimensional singularities (as seen in some liquid crystals)? (no)
5. Open problems?

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