1. (a) Show that if \( f : \mathbb{C} \rightarrow \mathbb{C} \) is holomorphic and \( \sup_{z \in \mathbb{C}} |Im f| < \infty \), then \( f \) is a constant.

(b) Does there exist a nonconstant holomorphic \( g \) on \( \mathbb{C} \setminus \{0\} \) with \( \sup_{z \in \mathbb{C} \setminus \{0\}} |Im f| < \infty \)? Give an example or explain why not.

2. Suppose \( X \) is a nonempty set, and \( f_n \) is a sequence of nonnegative real-valued functions on \( X \) converging uniformly to some function \( f : X \rightarrow \mathbb{R} \). Are the following 2 statements necessarily true?

(a) The sequence \( f_n^2 \) is uniformly convergent to \( f^2 \).

(b) The sequence \( \sqrt{f_n} \) is uniformly convergent to \( \sqrt{f} \).

Explain each answer with a proof or counterexample.

3. For \( a \in \mathbb{R} \), find
\[
\int_{-\infty}^{\infty} \frac{e^{iax}}{1 + x^2} \, dx.
\]

4. Suppose that \( f : [0, \infty) \rightarrow [0, \infty) \) is a continuous function in \( L^2 \).

(a) Show that if \( f \) is decreasing, then \( \lim_{x \to \infty} \sqrt{x} f(x) = 0 \).

Hint: Consider the integral of \( f^2 \) from \( a \) to \( 2a \).

(b) Show that, without the decreasing assumption, it is possible to have a continuous \( L^2 \) function \( f \) with \( \lim \sup_{x \to \infty} \sqrt{x} f(x) = \infty \).

5. Suppose \( D \) is the open unit disk \( \{z \in \mathbb{C} : |z| < 1\} \).

Find \( \int_{\partial D} \frac{f'(z)}{f(z)} \, dz \) where \( f(z) = z^5 - z + 2 \).

6. Suppose \( f \in L^1([0, 1]) \) and \( g(c) = \int_0^1 |f(t) - c| \, dt \) for \( c \in \mathbb{R} \)

(a) Show that \( \int_0^1 |f(t)| \, dt = \int_0^\infty \text{meas} \{t : |f(t)| > s\} \, ds \).

(b) Show that \( \lim_{c \to \pm \infty} g(c) = \infty \).

(c) Use (a) to show that \( g \) is absolutely continuous on \( \mathbb{R} \) and find a formula for \( g'(c) \) for a.e. \( c \in \mathbb{R} \).