(1) (10 points) Compute
\[ \int_0^\infty \frac{\cos 2x}{4 + x^2} \, dx . \]

(2) (5 points) Construct a subset \( K \) of \([0; 1]\) such that \( K \) is closed, \( K \) has positive Lebesgue measure, and \( K \) has empty interior.

(3) Denote \( \mathbb{D} = \{ z \in \mathbb{C} : |z| < 1 \} \) and \( Y = \{ z = x + iy \in \mathbb{C} : x \geq e^y \} \).
   (a) (5 points) Does there exist a non-constant analytic function \( f : \mathbb{D} \to Y \) such that \( f(0) = 1 \)?
   (b) (5 points) Does there exist a non-constant analytic function \( f : \mathbb{D} \to Y \) such that \( f(0) = 2 \)?
   (Justify your answers.)

(4) Consider the spaces \( L^p = L^p([0; 1], dm) \) where \( 1 < p < \infty \) and \( dm \) denotes Lebesgue measure, with the associated norm \( \| \cdot \|_p \).
   (a) (5 points) For \( 1 < q < p < \infty \) and \( f \in L^p \), show that \( f \in L^q \) and that \( \| f \|_L^q \leq \| f \|_{L^p} \).
   (b) (5 points) For \( f \in L^\infty \), show that \( \| f \|_{L^\infty} = \lim_{p \to \infty} \| f \|_p \).
   (c) (5 points) Prove or disprove: \( L^\infty = \bigcap_{1 \leq p < \infty} L^p \).

(5) Suppose \( f \in L^2 = L^2([0; 1]; dm) \) as in problem (4).
   (a) (5 points) Show that
   \[ F(x) = \int_0^x f(t) \, dm(t) \]
   is a continuous function on \([0, 1]\).
   (b) (5 points) Suppose that \( f_n, f \in L^2 \) and
   \[ \lim_{n \to \infty} \int_0^1 f_n(t) \, g(t) \, dm(t) = \int_0^1 f(t) \, g(t) \, dm(t) \]
   for all \( g \in L^2 \). Show that \( F_n \to F \) uniformly where \( F_n(x) = \int_0^x f_n(t) \, dm(t) \) for \( x \in [0; 1]\).

(6) (10 points) Show that, for any \( 0 < r < 1 \), there exists an integer \( n \geq 2 \) such that the polynomial
   \[ P_n(z) = 1 + 2z + 3z^2 + \cdots + nz^{n-1} \]
   has no roots in the disk \( \{ z \in \mathbb{C} : |z| < r \} \).