May 2010 - TOPOLOGY QUALIFYING EXAM - RICE UNIVERSITY

This is a 3 hour, closed book, closed notes exam. Show your work.

1. Let K be the Klein bottle.
(a) Identify the universal cover of K and describe generators for its group of covering transformations. Be as explicit as you can.
(b) Do the same for the cover $p : \tilde{K} \to K$ with $p_*(\pi_1(\tilde{K})) = [\pi_1(K), \pi_1(K)]$ – the commutator subgroup (that is, describe this covering space and describe generators for its group of covering transformations. Be as explicit as you can).

2. Describe $\pi_1(X)$ in each of the following cases
(a) $X = \mathbb{R}^3$ with the coordinate axes removed.
(b) $X = \mathbb{R}^4$ with the zw-plane ($x = y = 0$) and the xy plane ($z = w = 0$) removed.

3. Let X be the space obtained from a solid octagon by identifying sides as shown Figure 1 below.

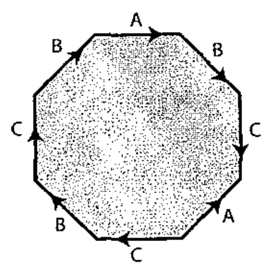

FIGURE 1

(a) Give a CW-structure for X (be careful with the vertices) and describe the cellular chain complex.
(b) Give a presentation for $\pi_1(X)$.
(c) Calculate $H_n(X; \mathbb{Z}_3)$ and $H^n(X; \mathbb{Q})$ for all $n \geq 0$.

4. Let X be a connected, orientable, 4-dimensional manifold without boundary such that $\pi_1(X) \cong \mathbb{Z}_{35}$ and $\chi(X) = 4$.
(a) Calculate $H_i(X; \mathbb{Z})$ for all i.
(b) Suppose $\widetilde{X}$ is a connected regular 5-fold covering space of X. Calculate $H_i(\widetilde{X}; \mathbb{Z})$ for all i.

5. Prove that, for $n \geq 2$, any continuous map $f : \mathbb{CP}(n) \to S^2$ induces the zero map on $H_2(-; \mathbb{Z})$.