This is a 3 hour, closed book, closed notes exam. For maximum credit include justification for all steps. Sign the Rice honor pledge at the end of your exam.

1. Let \( f : S^n \to S^n \) be a smooth map with no fixed points. Prove that the degree of \( f \) is \((-1)^{n+1}\).

2. Let \( L_1 \) and \( L_2 \) be disjoint straight lines in \( \mathbb{R}^3 \). Calculate \( \pi_1(\mathbb{R}^3 \setminus (L_1 \cup L_2)) \).

3. Let \( W = S^1 \vee S^1 \). Construct three connected 3-fold covers of \( W \) that are distinct up to covering space equivalence, including at least 1 irregular cover. For each of these three covers, describe the covering map, say whether or not the cover is regular, and give the corresponding subgroup of \( \pi_1(W) \).

4. For \( n \geq 2 \), prove that for any continuous map \( f : \mathbb{C}P^n \to S^2 \), the induced map \( f_* : H_2(\mathbb{C}P^n; \mathbb{Z}) \to H_2(S^2; \mathbb{Z}) \) is the zero map.

5. Give an example (a CW complex) for each of the following or state that such an example does not exist. Give a brief justification in all cases.

   (a) Two spaces with isomorphic \( \pi_1 \) but non-isomorphic integral homology groups.

   (b) Two spaces with isomorphic integral homology groups but non-isomorphic \( \pi_1 \) (give \( \pi_1 \) of the spaces).

   (c) Two spaces with isomorphic integral homology groups but non-isomorphic cohomology groups.

   (d) Two spaces that are homotopy equivalent but not homeomorphic.

   (e) Two spaces with isomorphic \( \pi_1 \) and isomorphic integral homology groups that are NOT homotopy equivalent.

6. Let \( M \) be a compact contractible \( n \)-manifold with boundary. Prove that \( \partial M \) is a homology \((n - 1)\)-sphere, i.e. that \( H_i(\partial M; \mathbb{Z}) \cong H_i(S^{n-1}; \mathbb{Z}) \) for all \( i \).