1. Let \( W = S^1 \vee S^1 \) be the wedge of 2 circles. Describe four distinct connected 3-fold covering spaces of \( W \) including at least one irregular cover. In each case, give the group of covering transformations, say whether or not the covering is regular and give the corresponding subgroup of \( \pi_1(W) \).

2. Let \( S \) and \( S' \) be surfaces of genus 2 as shown in the figure. Let \( X \) be the space obtained from \( S \cup S' \) by identifying the circle \( \gamma \) in \( S \) to the circle \( \gamma' \) in \( S' \).

![Diagram of surfaces S and S']

a) Give a presentation for \( \pi_1(X) \).

b) Compute \( H_p(X) \) for all \( p \).

3. Let \( W = S^1 \vee S^1 \vee S^1 \) as shown below. Let \( x, y, z \) be the three loops indicated going around the first, second, third circles respectively. Let \( X = W \cup f_1 e_1^2 \cup f_2 e_2^3 \) be the space obtained from \( W \) by adjoining one 2-cell via the map \( f_1 \) which forms the loop \( xyx^{-1}zy^{-1} \), and another 2-cell via the map \( f_2 \) which forms the loop \( z^7 \).

![Diagram of loop x, y, z]

a) Give a cellular chain complex for \( X \) (including the boundary maps).

b) Compute \( H_p(X; \mathbb{Z}_2) \) for all \( p \).

c) Compute \( H^p(X; \mathbb{Q}) \) for all \( p \).

4. The following two problems are independent.

a) Suppose \( A \subset X \). Define \( H^p(X, A) \).

b) Calculate \( H_p(S^1 \times D^2, S^1 \times \partial D^2) \) for all \( p \).

5. Let \( X \) and \( Y \) be compact, connected, oriented \( n \)-dimensional manifolds without boundary and let \( f : X \to Y \) be a continuous map. Suppose \( \beta_p(X) < \beta_p(Y) \) for some \( p > 0 \).

a) Prove that \( f^* : H^p(Y; \mathbb{Q}) \to H^p(X; \mathbb{Q}) \) has a non-trivial kernel.

b) Show that \( f \) is a degree zero map.