This is a 4 hour, closed book, closed notes exam. Justify all of your work, as much as time allows. Write and sign the Rice honor pledge at the end of the exam.

Honor pledge: On my honor, I have neither given nor received any unauthorized aid on this exam.

1. Let \( r : X \to A \) be a retract and \( X \) be a contractible space. Prove that \( A \) is contractible.

2. Let \( M \) be a closed, smooth, simply-connected \( n \)-dimensional manifold and let \( T = S^1 \times \cdots \times S^1 \) be the \( n \)-dimensional torus. Prove that there does not exist a smooth immersion from \( M \) to \( T \).

3. Let \( W = S^1 \vee S^1 \) be the wedge of 2 circles. Describe four distinct connected 3-fold covering spaces of \( W \) including at least one irregular cover. In each case, give the group of covering transformations, say whether or not the covering is regular and give the corresponding subgroup of \( \pi_1(W) \).

4. Let \( H \) be a solid handlebody of genus 2. Recall that \( H \) can be obtained as follows. Let \( S \) be the surface (with boundary) pictured in Figure 1, then \( H = S \times I \). \( H \) can also be viewed as a 3-dimensional manifold obtained by thickening up a wedge of circles (its boundary is a genus 2 surface).

Let \( f : H \to \mathbb{R}^3 \) be a topological embedding. Let \( X = \text{int}(f(H)) \) be the image of \( f \) in \( \mathbb{R}^3 \). Compute \( H_p(\mathbb{R}^3 - X) \) for all \( p \).

![Figure 1. S](image)

5. Suppose \( Y \) is a topological space which is obtained from the union of a 2-sphere \( S^2 \) and a torus \( T \) by identifying the circle \( A \) to the circle \( A' \) and the circle \( B \) to the circle \( B' \) as shown below. Thus \( S^2 \cap T \cong S^1 \sqcup S^1 \).

![Diagram](image)

a) Calculate \( H_p(Y; \mathbb{Z}) \) for all \( p \).

b) Sketch or describe "geometric" representatives of the generators of \( H_1(Y; \mathbb{Z}) \) and \( H_2(Y; \mathbb{Z}) \).

c) Calculate \( \pi_1(X) \).

d) Sketch or describe a connected 2-fold covering space of \( Y \) and the covering map.
6. Let $M$ be a closed, connected, oriented 4-dimensional manifold with $b_2(M) \geq 1$. Prove that any continuous map $f : S^4 \to M$ has degree 0.