Undergraduate Mathematics Colloquium

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"From Knots to Hyperbolic 3-manifolds"

Monday October 18

4:00pm in Herman Brown 227

ABSTRACT: The mathematical study of knots has a rich history, dating back to 1771 (happy 250th birthday, knot theory!). Though there are many interesting questions one can ask about knots, the fundamental question of the field is to determine whether two knots (that is, knotted-up, embedded circles in R^3) are equivalent, i.e., whether one can be continuously deformed into the other. Though this question is quite hard, one surprisingly effective strategy is to study the space around the knots, rather than the knots themselves. This space (the "knot complement") is far more complicated than the knot, but often carries a rich geometric structure that can be used to differentiate between two knots. In the 70's, Thurston demonstrated a way to cut apart this space into tetrahedra, and then to determine the geometric structure by solving a system of equations. The goal of this talk will be to describe Thurston's decomposition of a knot complement into tetrahedra, using 3D modeling and animation to visualize the process for a particular example. We will also give an idea of how this decomposition can be used to write down a system of equations whose solution gives the knot complement the structure of a 3-dimensional hyperbolic manifold. In the last few minutes, we'll use the computer program SnapPy to see what the inside of a hyperbolic 3-manifold looks like.

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