Parameterizations to Know

Curves

1. Circle of radius $r$ centered at the origin in the $xy$-plane:
   $$ x = r \cos \theta, \quad y = r \sin \theta, \quad 0 \leq \theta \leq 2\pi $$
   Orientation: Counterclockwise

2. Ellipse given by $x^2/a^2 + y^2/b^2 = 1$ in the $xy$-plane:
   $$ x = a \cos \theta, \quad y = b \sin \theta, \quad 0 \leq \theta \leq 2\pi $$
   Orientation: Counterclockwise

3. Line Segment from $P_0$ to $P_1$:
   Let $r_0$ and $r_1$ denote the position vectors for $P_0$ and $P_1$ respectively. Then the line segment is given by
   $$ \vec{r}(t) = (1-t)\vec{r}_0 + t\vec{r}_1, \quad 0 \leq t \leq 1. $$
   Alternatively, let $P_0 = (x_0, y_0, z_0)$ and let $\overrightarrow{P_0P_1} = (a, b, c)$. Then the line segment is given by
   $$ x = x_0 + at, \quad y = y_0 + bt, \quad z = z_0 + ct, \quad 0 \leq t \leq 1. $$
   Orientation: From $P_0$ to $P_1$.

4. General function $f(x) = y$, $a \leq x \leq b$ in the $xy$-plane:
   $$ x = x, \quad y = f(x), \quad a \leq x \leq b $$
   Orientation: From the point $(a, f(a))$ to the point $(b, f(b))$, i.e. in the direction of increasing $x$-value.

Surfaces

1. Sphere of radius $a$ centered at the origin:
   $$ x = a \sin \phi \cos \theta, \quad y = a \sin \phi \sin \theta, \quad z = a \cos \phi, \quad 0 \leq \phi \leq \pi, \quad 0 \leq \theta \leq 2\pi $$
   Orientation: $r_\phi \times r_\theta = \sin^2 \phi \cos \theta \hat{i} + \sin^2 \phi \sin \theta \hat{j} + \sin \phi \cos \phi \hat{k}$ points outward

2. Surfaces of revolution:
   Let $y = f(x)$, $a \leq x \leq b$ and rotate it around the $x$-axis. Then we can parameterize the resulting surface by
   $$ x = x, \quad y = f(x) \cos \theta, \quad z = f(x) \sin \theta, \quad a \leq x \leq b, \quad 0 \leq \theta \leq 2\pi. $$
   Orientation: $r_x \times r_\theta$ points outward

3. General function $z = f(x, y)$, for $(x, y)$ in some closed, bounded domain $D$:
   $$ x = x, \quad y = y, \quad z = f(x, y), \quad (x, y)$ in $D $$
   Orientation: $r_x \times r_y = -f_x \hat{i} - f_y \hat{j} + \hat{k}$ points outward