Math 463/563 - Midterm 1
Due: Monday, 10/2/17 by 5pm

Instructions: You have three hours to complete the exam, although you may complete it in two sittings. You may not consult your notes, the text, or any other reference, nor may you discuss the exam with others. Your completed exam should be turned in by the date and time above, either by putting it in the envelope on my door or e-mailing it to me.

Problems:

1. Let $R$ be a ring and let $I \subseteq R$ be an ideal with the property that every element of $R$ that is not in $I$ is a unit of $R$. Prove that $I$ is the unique maximal ideal of $R$.

2. Let $F$ be a field of characteristic 2. Prove that the idempotents of $F$ form a ring.

3. Let $R$ be a ring and suppose that for all $a \in R$ there exists $n > 1$ such that $a^n = a$. Show that every prime ideal of $R$ is maximal.

4. Let $R$ be a ring of characteristic $n$. If $n = 0$, show that $R$ contains a subring isomorphic to $\mathbb{Z}$. If $n > 0$, show that $R$ contains a subring isomorphic to $\mathbb{Z}/n\mathbb{Z}$. Deduce that if $F$ is a field, then $F$ has characteristic 0 or $p$, where $p$ is prime.

5. Let $R$ be a principal ideal domain. Show that every proper ideal of $R$ is contained in a maximal ideal.