Math 463/563 - Homework 8

Due: Thursday, 10/26 by 5pm

Part A: All Students

Hand in your solutions to the following exercises from Chapter 14 of Artin's book.

7.5, 7.9, 8.6*

* This problem is vaguely stated, so to be precise, classify the primes in $\mathbb{C}[\epsilon] = \mathbb{C}[x]/(x^2)$ and use these to give a more specific structure theorem for these modules. (Recall, in PIDs that prime elements (up to units) are in 1-1 correspondence with prime ideals, that irreducible is the same as prime, and that maximal ideals are generated by irreducible elements. Now use the correspondence theorem to prove a precise classification of the maximal ideals of $\mathbb{C}[\epsilon]$).

In addition, please hand in your solutions to the following exercises:

1. Let R be any ring, let $A_1, \dots, A_m$ be R-modules and let B_i be a submodule of A_i, $1 \leq i \leq m$. Prove that
$$(A_1 \oplus \cdots \oplus A_m)/(B_1 \oplus \cdots \oplus B_m) \simeq (A_1/B_1) \oplus \cdots (A_m/B_m).$$

2. Let R be a PID, let a be a nonzero element of R, and let $M = R/(a)$. For any prime p of R prove that
$$p^{k-1}M/p^kM \simeq \begin{cases} R/(p) & \text{if } k \leq n \\ 0 & \text{if } k > n, \end{cases}$$
where n is the largest power of p dividing a in R.

   (Hint: When proving two quotients are isomorphic, it is useful to recall the corollary of the correspondence theorem.)

Part B: Math 563 Students

Read Section 2.6 of Serre's book. There are no problems on this section due this week, but please do the reading.