Practice Problems

1. Evaluate the following Integral using Cauchy's theorem.

(a) $\displaystyle\int_{|z|=1} \frac{z\,dz}{(z-2)^2}$ (b) $\displaystyle\int_{|z|=2} \frac{z^2+1}{z-i}\,dz$

2. Evaluate $\displaystyle\int_C \bar{z}^2\,dz$ around the triangle with vertices at $z = 0, 2, 2+2i$. Here C is closed contour. Why does not Cauchy's Theorem apply to this integral?

3. Use Fundamental theorem of Calculus to evaluate the following integrals.

(a) $\displaystyle\int_{1-\pi i}^{2+3\pi i} e^{-2z}\,dz$ (b) $\displaystyle\int_0^{\pi i} \sinh 5z\,dz$.

4. Use Cauchy's integral formula to evaluate the following definite integrals:

(a) $\displaystyle\int_0^{2\pi} \frac{d\theta}{13-12\cos\theta}$ (b) $\displaystyle\int_0^{\pi} \frac{d\theta}{1+\sin^2\theta}$

(c) $\displaystyle\int_{|z|=2} \frac{e^z}{(z+1)(z-3)^2}\,dz$

5. Evaluate the following integrals.

(a) $\displaystyle\int_{1-i}^{1+i} \frac{dz}{z}$, Γ in $\operatorname{Re} z > 0$

(b) $\displaystyle\int_{-1}^{1} e^{z}\,dz$, Γ is semicircle through i.

6. Find and classify the isolated singularities.

(a) $\tan z$ (b) $\cos\left(1 - \frac{1}{z}\right)$

7. True or False:

(i) If $f(z)$ has a pole of order m at $z=0$ then $f(z^2)$ has a pole of order $2m$ at $z=0$

(ii) If $f(z)$ has a zero of order m at z_0 and $g(z)$ has a pole of order n $(n<m)$ at z_0, then the product $f(z)g(z)$ has a removable singularity at z_0.

8. Find the Laurent series

(a) $z^{-3/2}\sinh\sqrt{z}$, about $z=0$

(b) $z^{-1}\cosh z^{-1}$, about $z=0$

(c) $\dfrac{z-\sin z}{z^3}$, about $z=0$

(d) $\dfrac{z}{(z-1)(z-2)}$, (i) $0<|z-2|<1$

(ii) $|z-2|>2$.

9. Compute the poles and Residues at poles.

9. (a) $\dfrac{1-e^{2z}}{z^4}$ (b) $\dfrac{e^z}{z^2+\pi^2}$

10. Residue theorem:

(a) $\displaystyle\int_C \frac{z}{1+2z}\,dz$, C denote the square whose sides lie along the lines $x=\pm 2$, $y=\pm 2$.

(b) $\displaystyle\int_0^\infty \frac{x^2\,dx}{(x^2+1)^2}$

(c) $\displaystyle\int_0^{2\pi} \frac{\cos 3\theta}{5+4\cos\theta}\,d\theta$

(d) $\displaystyle\int_{|z|=1} \frac{\sin z - \sinh z}{z^8}\,dz$

HW. (e) $\displaystyle\int_0^\infty \frac{1+x^2}{1+x^4}\,dx$

HW. (f) $\displaystyle\int_{-\infty}^\infty \frac{dx}{\cancel{1+x^2+x^4}}$ 1+x^2+x^4

HW. (g) $\displaystyle\int_0^\infty \frac{\cos x}{4+x^2}\,dx$