QUALIFYING EXAM SYLLABUS

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September 9th, 2005.

Major Topic: Algebraic Number Theory

Number fields: Dedekind Domains, rings of integers, norm and trace, discriminant and different, Galois extensions, prime factorization in extension fields, ideal class group (including finiteness of the class number), Dirichlet Unit theorem. Quadratic and cyclotomic extensions. Decomposition and Inertia Groups.
Local fields: Hensel's lemma and Henselian fields, completions and valuations, extensions of valuations. Unramified, tamely ramified and totally ramified extensions.
Class field theory: Statements of class field theory, including Artin Reciprocity and the Chebotarev Density theorem. Ideles, adeles.

References:
Chapters 1 & 2 §1–9 of
[1] Neukirch, J. *Algebraic Number Theory* Grund. der Math. Wiss. 322 Springer, Berlin, 1999.
Chapters VI & VII of
[2] Cassels & Fröhlich, A. (Eds.), *Algebraic Number Theory* Academic Press, London, 1967.

Major Topic: Algebraic Geometry

Schemes: Affine, irreducible, reduced, noetherian. Fibered Products.
Morphisms: separated, proper (valuative criteria), finite, finite type, closed immersions. Projective morphisms.
Sheaves of modules: Quasi-coherent, Coherent. Twisting sheaf.
Divisors Weil, Cartier divisors. Picard group. Invertible sheaves. Linear systems.
Differentials: Differentials, sheaves of differentials.
Cohomology: Cohomology of sheaves. Cech cohomology, Cohomology of projective spaces. Grothendieck's vanishing theorem (statement only); Serre's criteria for affineness.
Curves: Statement of Riemann-Roch for nonsingular projective curves over an algebraically closed field.

Reference:
Chapter 2 §1–6, §7 (p.149–160), §8 (p.172–184), Chapter 3 §1–5, Chapter 4 §1 of
[1] Hartshorne, R. *Algebraic Geometry* Springer, New York, 1977.

Minor Topic: Complex Analysis

Holomorphic and meromorphic functions: Taylor and Laurent series. Linear Fractional Transformations. Liouville's theorem, fundamental theorem of algebra. Rouche's theorem.
Complex integration: Cauchy's theorem, Cauchy's integral formula, Morera's theorem, residue theorem. Harmonic functions.
Product Developments Weierstrass products, Hadamard's Theorem.
Elliptic funtions: Weierstrass $\wp$-function.

Reference:
Chapters 1–4, Chapter 5 §1–3, Chapter 7 §3 of
[1] Ahlfors, L. *Complex Analysis*, 3rd edition, McGraw–Hill, New York, 1979.