3. Let C be the space curve parametrized by $x(t) = t^2, y(t) = t, z(t) = t^2, 0 \leq t \leq 1$.

   (a) DIRECTLY calculate the line integral $\int_C \mathbf{f} \cdot d\alpha$ if $\mathbf{f}(x, y, z) = (yz,\ xz,\ xy)$.

   (b) Calculate the same integral $\int_C \mathbf{f} \cdot f\alpha$ using the Fundamental Theorem of line integrals.

Problem 7 Evaluate the integral $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$, where S is the part of the sphere $x^2 + y^2 + z^2 = 1$ with $z \geqslant 0$, and $\mathbf{F}(x, y, z) = (z - y)\mathbf{i} + \frac{x}{x^2+y^2}\mathbf{j} + e^{xyz}\mathbf{k}$. Choose the upward orientation for S.

8. Every curve in this problem is oriented in the counterclockwise direction. Apply Green's Theorem in the following calculations.

   (a) Let C_1 be the square with vertices $(1, 1)$, $(-1, 1)$, $(-1, -1)$ and $(1, -1)$ and $\mathbf{f_1} = (x + y^2, y + x^2)$, calculate $\int_{C_1} \mathbf{f_1} \cdot \vec{\mathbf{T}}\, ds$.

   (b) Let C_2 be the circle $x^2 + y^2 = 9$ and $\mathbf{f_2} = (y^2 - x^2, x^2 + y^2)$, calculate the flux $\int_{C_2} \mathbf{f_2} \cdot \vec{\mathbf{n}}\, ds$.

9. Let B be the region bounded by $x + 2y = \pi$, $x + 2y = 0$, $x - 2y = 0$, and $x - 2y = \pi$. Use change of variable formula to rewrite the following integral in (u, v) variables so that all the integration limits are constant numbers. Evaluate the integral.

$$\int\int_B (x - 2y)^2 \sin(x + 2y) dA = \int_?^? \int_?^? (?) du dv$$

4. (a) Let $\mathbf{f} = (P, Q, R) = (x + z, ax + y, bx + cy - z)$ be a vector field where a, b, and c are constants. For what value(s) of a, b, and c is $\mathbf{f}$ a gradient field?

   (b) Find a potential function for the vector field $\mathbf{g} = (\ln y, x/y, \sin z)$ on the set $\{(x, y, z) \in \mathbb{R}^3, y > 0\}$.

6. Convert the following integral into an integral in polar coordinate system and evaluate it.

$$\int_0^2 \int_x^{\sqrt{8-x^2}} (x^2 + y^2)^{\frac{3}{2}} dy dx = \int_?^? \int_?^? (?) dr d\theta$$

Answer:

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^{2\sqrt{2}} r^4 dr d\theta = \frac{\pi}{4}\frac{1}{5} r^5 |_0^{2\sqrt{2}} = \frac{32\sqrt{2}\pi}{5}$$

7. Let L represent the line segment joining points $(0, 0, a)$ and $(0, b, 0)$ for $a > 0$ and $b > 0$. Rotate L about the z axis to obtain a surface. Sketch the surface. Set up a double integral to compute the area of the surface and evaluate it.