Final Problem set Math 444

1. Let $X = S^1 \times \mathbb{C}P(2)$. Describe all of the connected covering spaces of X (up to isomorphism).

2. Let X be the wedge (one point union) of 2 circles $S^1 \vee S^1$. Describe at least 4 distinct connected 3-fold covers of X, including at least one irregular cover and one regular(normal) cover (say which ones are which). Compute the groups of deck transformations for each.

3. a) Discuss all connected covering spaces of $S^1 \times \mathbb{RP}(3)$. Specifically discuss how many there are, regular/irregular, how you know there aren't any more, their groups of covering translations, possibly include pictures. Beware not every subgroup of $A \times B$ is a "diagonal" subgroup.

b) Calculate $\pi_2(S^1 \times \mathbb{RP}(3))$.

4. Let $p : \tilde{X} \to X$ be a simply-connected covering space of X and let $A \subset X$ be a path-connected, locally path-connected subspace and $\tilde{A}$ a path component of $p^{-1}(A)$. Show that $p : \tilde{A} \to A$ is the covering space corresponding to the kernel of the map $\pi_1(A) \to \pi_1(X)$ induced by inclusion .

5. For which compact surfaces Σ is it true that $\pi_1(\Sigma)$ has a proper subgroup H that is isomorphic to $\pi_1(\Sigma)$? (you may assume that any noncompact surface is homotopy equivalent to a wedge of circles) (Hint: use behavior of Euler characteristic under coverings).

6. Problem 12-1 from book.