First-year and **second**-year grad students should do all of these problems and hand them in by next Thursday. Other graduate students should do and hand in numbers $2, 5$ and 6, and whatever other problems would benefit them.

PROBLEM SET 3**: MATH** 541 **FALL** 2009

1. Prove, using the geometric definition of the Blanchfield form of a knot, that the Blanchfield form is Λ linear in the first variable and Λ conjugate-linear in the second variable. Use that, since t acts by an orientation preserving homeomorphism on $\widetilde{S^3 - K}$, $tx \cdot ty = x \cdot y$ where $\cdot$ is the intersection form.
2. Find knots K_1 and K_2 that have isomorphic Alexander modules, but non-isomorphic Blanchfield forms (in fact are not even equivalent in the Witt group (the knots are not equivalent in alg. concordance group). Hint:it is easy once you know the answer. Find knots K_3 and K_4 that have non-isomorphic Blanchfield forms but whose forms are equivalent in the Witt group. Finally find (seemingly) distinct knots K_5 and K_6 that have isomorphic Blanchfield forms.
3. If $(\mathcal{M}, \lambda) \in W(R, S)$ show that $(\mathcal{M}, -\lambda)$ is its inverse where $(-\lambda)(x, y) = -(\lambda(x, y))$.
4. Using the formula for the Blanchfield form in terms of the Seifert matrix, calculate $\mathcal{B}\ell_T(1, 1)$ where the T is the right-handed trefoil knot (a twist knot) and 1 is a generator. Compare answers with friends. Is it possible that answers look different? Why?
5. Suppose K is a knot whose Alexander polynomial is not divisible by any symmetric prime. Show that K is algebraically slice.
6. Suppose that $f : X \to Y$ is an orientation-preserving homotopy equivalence between closed, connected, oriented $2k$ manifolds. Show that the intersection forms on $H_k(X; \mathbb{Z})$ and $H_k(Y; \mathbb{Z})$ are isomorphic. Use the definition on page IV$-$1. You might need to look up a definition of Poincaré duality that is functorial. Deduce that $\mathbb{CP}(2)$ is homotopy equivalent to $-\mathbb{CP}(2)$ but not not orientation-preserving homotopy equivalent.