

Second Midterm Exam  
Math 212 Fall 2010

**Instructions:** This is a **90 minute** exam. You should work alone, without access to any book or notes. No calculators are allowed. Do not discuss this exam with anyone other than your instructor. When you have completed the exam, write out and sign the Honor Code pledge on the front.

The exam consists of 6 questions. You must show all of your work on each problem to receive full credit, and **be sure to clearly indicate your final answer** to each question.

Name:

Write out the Honor Pledge:

Signature:

Problem	Score
1	/15
2	/15
3	/20
4	/15
5	/20
6	/15
Total	

1. [15 Points] Let  $\mathbf{f}(x, y, z) = (xz, \sin(xy))$  and  $\mathbf{g}(u, v) = (u^2, e^{uv}, \ln v)$

(a) Compute the derivatives  $\mathbf{Df}$  and  $\mathbf{Dg}$ .

(b) Using the multivariable chain rule, compute the derivative  $\mathbf{D}(\mathbf{f} \circ \mathbf{g})$ . (Your final answer should be a matrix whose entries are functions of  $u$  and  $v$ .)

- (c) Using the multivariable chain rule, compute the derivative  $\mathbf{D}(\mathbf{g} \circ \mathbf{f})$ . (Your final answer should be a matrix whose entries are functions of  $x$ ,  $y$ , and  $z$ .)

2. [20 Points] Let  $f(x, y) = 2x^2 + y^4 - 4xy$ .

(a) Find the critical points of  $f$ .

(b) Compute the Hessian matrix of  $f$ .

(c) Using the multivariable second derivative test, classify the critical points of  $f$ .

(d) Starting from the point  $(3, 2)$ , in which direction does the function  $f$  decrease most rapidly?

3. [15 Points] Let  $I = \int_0^4 \int_{\sqrt{x}}^2 \frac{x}{y^5 + 1} dy dx$

(a) Write  $I$  as the double integral over a region  $D$  in the plane. Sketch the region  $D$ .

(b) Reverse the order of integration, i.e. rewrite  $I$  as an iterated integral “ $dx dy$ ”.

(c) Compute  $I$ .

4. [15 Points] Find the dimensions of a right circular cylinder of the maximum possible volume that can be inscribed in a sphere of radius 16. (Make sure to clearly explain what your variables mean.)

5. [20 Points] Let  $W$  be the region in  $\mathbb{R}^3$  defined by the inequalities  $y \geq x^2$ ,  $z \leq 3 - y$ , and  $z \geq 0$ . Let  $f(x, y, z)$  be a continuous function on  $W$ .

(a) Sketch the region  $W$ .

(b) Write  $\iiint_W f(x, y, z) dV$  as an iterated integral in the order “ $dz dy dx$ ”.

(c) Write  $\iiint_W f(x, y, z) dV$  as an iterated integral in the order “ $dx dy dz$ ”.

(d) Evaluate  $\iiint_W y dV$

6. [15 Points] Let  $f(x, y) = x^2 + y^2 - 2x - 4y$ . Find the absolute maximum and minimum of  $f$ , subject to the constraints  $x^2 + y^2 \leq 2$  and  $y \geq 0$ .