MATH 468
Potpourri
Spring 2014
Frank Jones
MWF 9-9:50 SST 106

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Text: Graham, Knuth, & Patashnik, *Concrete Mathematics, 2nd ed*, Addison-Wesley

Goal: To learn as much about this excellent book as possible, and at the same time to develop a definite *joy* in accomplishing prodigious mathematical feats.

Prerequisites: MATH 102. This modest prerequisite comes with the observation that the course has a 400 number. This course will really stretch and thus strengthen your (*our*) calculus skills!

Homework: Homework will be due intermittently. I *strongly* encourage you to work together as much as you desire.

Exams and grading: Several of the HW assignments will be called "exams," and I shall grade those. Your final grade will represent your progress throughout the semester.

Exercises: There are over 500 in the book. As the cover states, "Complete answers are provided for all exercises, except research problems, making the book particularly valuable for self-study."

That last sentence presents quite a challenge for me! I have to find a way to assess your progress and prowess, but all the answers to all the exercises are in the book. And this is not supposed to be a "self-study" course. I think we'll be able to come to some clear conclusions about your individual understanding.

Meanwhile, I offer four exercises that may help get you in the right frame of mind for the semester.

A. Define $x_0 = 1$ and for $n \geq 0$ let $x_{n+1} = 3x_n + \lfloor x_n\sqrt{5} \rfloor$. Find an explicit formula for x_n for all n.
(Putnam Competition 2007)
This exercise is due Monday February 24.

B. Look at the Fibonacci sequence as defined on page 291. Compute the infinite product of the quotients $(F_{2n} + 1)/(F_{2n} - 1)$ for n=2,3,4, ... (You will probably want to approach this as follows:

a. Compute a few finite products, and guess a pattern that holds for every finite product of terms for n = 2, 3, 4, ..., N.
b. Prove that your pattern is actually valid for all N.
c. Finally, let $N \to \infty$

I saw this problem in *Mathematics Magazine*, vol. 79, No. 5, December 2006, p. 393.)
This exercise is due Monday March 17.

C. Exercise 54 in Chapter 7 (page 380):

54 Consider the following curious construction:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	...
1	2	3	4		6	7	8	9		11	12	13	14		16	...
1	3	6	10		16	23	31	40		51	63	76	90		106	...
1	3	6			16	23	31			51	63	76			106	...
1	4	10			26	49	80			131	194	270			376	...
1	4				26	49				131	194				376	...
1	5				31	80				211	405				781	...
1					31					211					781	...
1					32					243					1024	...

(Start with a row containing all the positive integers. Then delete every mth column; here $m = 5$. Then replace the remaining entries by partial sums. Then delete every $(m-1)$st column. Then replace with partial sums again, and so on.) Use generating functions to show that the final result is the sequence of mth powers. For example, when $m = 5$ we get $\langle 1^5, 2^5, 3^5, 4^5, \ldots \rangle$ as shown.

The solution given on page 574 is extremely cryptic, but I will want all details supplied. You must present a complete proof with all details given. (You are not required to use GKP's proof.) <u>This exercise is due the last week of class.</u>

D. My daughter has a puzzle which consists of a board with 7 holes in a line, 3 white pegs, and 3 black pegs. At the start, these are arranged in the line WWW – BBB. One is supposed to move them to the state BBB – WWW, using two types of moves: slide a peg from one hole to an adjacent one, or jump over one peg to land in the empty hole. (For instance, the first move might result in WW – WBBB or it might result in W – WWBBB.) This is easy enough to accomplish, and I can do it in 15 moves. The question for you is this: for any positive integer n, suppose you have a puzzle with n white pegs, n black pegs, and a board with $2n+1$ holes. Determine, *with proof*, the smallest number of moves required to switch the white and black pegs.

<u>This exercise is essentially a research problem. I hope some of you can do it.</u> (No one in the two previous classes on this subject managed to solve it.)

(I am quite sure what the answer is, but I am unable to verify it.)