MATH 468 Handout # 15 April 14, 2014

EXAM 3

Due Monday April 21

1. Find (with proof) the largest denominator in row n of the Stern-Brocot tree.

2. Do the Tower of Hanoi problem which appears in the **Basic** exercise 23 of Chapter 4.

3. Recall from HW 6 the definition of Motzkin paths. For the current problem let P_n denote the number of Motzkin paths with a total of n steps of the form (1,1) or (1,0) and which have no peaks: meaning that a step (1,-1) cannot immediately follow a step (1,1). As usual, define $P_0 = 1$.

Find the generating function for this sequence.

4. Similarly, let Q_n denote the number of paths from (0,0) to (2n,0) which never go below the horizontal axis and which use only the steps (1,1) and (2,0) and (1,-1).

Find the generating function for this sequence.

5. Solve **Exam problem** 71 in Chapter 5:

71 Let

$$S_n = \sum_{k\geqslant 0} \binom{n+k}{m+2k} a_k,$$

where m and n are nonnegative integers, and let $A(z) = \sum_{k\geqslant 0} a_k z^k$ be the generating function for the sequence $\langle a_0, a_1, a_2, \ldots \rangle$.

a Express the generating function $S(z) = \sum_{n\geqslant 0} S_n z^n$ in terms of $A(z)$.

b Use this technique to solve Problem 7 in Section 5.2.