MATH 468 Handout #8 February 26, 2014

HW 6: due Wednesday March 12

This concerns the **Motzkin numbers** M_n , which I introduced in Handout #7.

1. How many ways are there to connect some pairs of n points on a circle with nonintersecting chords? ("Some" may include "none.") Let M_n denote the number of ways.

Recall that $M_0 = 1$, $M_1 = 1$, $M_2 = 2$, $M_3 = 4$, $M_4 = 9$, $M_5 = 21$, $M_6 = 51$ . . . .

Establish a recursion relation for this sequence, and find the generating function.

2. Let X_n denote the number of *Motzkin paths* from (0,0) to (n,0). These are paths which never go below the horizontal axis and which use only the steps (1,1) and (1,0) and (1,-1).

Establish a recursion relation for this sequence, and conclude that $X_n = M_n$ for all n.

3. (Optional) The fact that $X_n = M_n$ implies the *existence* of a bijection between the geometries described in **1** and **2**. Can you find a "natural" bijection, one which relies only on the geometries involved in the two scenarios?