Homework 3, NOW due Tuesday, Sept.23

2.5, # 5 (Hint: recall # 3).

Extra Problem 1. Show that any first partial derivative of a harmonic function is harmonic.

Extra Problem 2. Show that the sum of 2 subharmonic functions is subharmonic.

Extra Problem 3. Show how to use Problems 1 and 2 to prove 2.5 4(d). (Maybe you used this proof in HW2)

Extra Problem 4. (a) For what constants A, B, C, D, E, F is the polynomial

$$P(x, y) = Ax^2 + Bxy + Cy^2 + Dx + Ey + F$$

a harmonic function on $\mathbf{R}^2$.

(b) Find a harmonic polynomial of degree 3.

Extra Problem 5. Suppose $u(r, \theta)$ is the polar coordinate expression for a harmonic function on the plane. What PDE (in terms of the r and θ partial derivatives does u satisfy?

Hint: part of the answer is derived in 2.2.1(a)

Using this PDE, find which positive integers m, n make $r^m \sin(n\theta)$ harmonic?