Homework 3, due Tuesday, Oct.28

2.5, # 17(i) Compute $\frac{d}{dt}[k(t) + p(t)]$. Show how this conservation of total energy implies that this solution of this initial-value, initial-velocity problem for the wave equation is *unique*.

Extra Problem 1. Suppose now that, for $\alpha > 0$, u is a solution of the *dispersion equation* $u_{tt} - u_{xx} + \alpha\, u_t = 0$ with the same initial data $u(x,0) = g(x), u_t(x,0) = h(x)$. Show that now the total energy $k(t) + p(t)$ is nonincreasing. Now show that this solution of this initial-value, initial-velocity problem for the dispersion equation is *unique*.

Extra Problem 2. Let Ω be the rectangle $(0,1) \times (0,2)$ in the $X - Y$ plane. For a fixed number $\lambda \geq 0$ consider the problem of finding a solution u of

$$\begin{aligned} u_{xx} + u_{yy} + \lambda\, u &= 0 \quad in \quad \Omega\ , \\ u(0,y) \ = \ 0 &= u(1,y) \quad for \quad 0 \leq y \leq 2\ , \\ u(x,0) \ = \ 0 &= u(x,2) \quad for \quad 0 \leq x \leq 1\ . \end{aligned}$$

First find all λ such that there is a solution (other than the 0 function) in the form $X(x)Y(y)$ for some functions $X(x)$ and $Y(y)$. For each such fixed λ find all the corresponding solutions $X(x)Y(y)$.

Extra Problem 3. Now consider the 2d wave equation $u_{tt} - u_{xx} - u_{yy} = 0$ for $u(x,y,t)$ with for (x,y) in the above rectangle Ω and $t > 0$. Find all solution in the form $X(x)Y(y)T(t)$ satisfying the zero boundary conditions:

$$\begin{aligned} u(0,y,t) \ = \ 0 &= u(1,y) \quad for \quad 0 \leq y \leq 2\ ,\ t > 0\ , \\ u(x,0,t)) \ = \ 0 &= u(x,2) \quad for \quad 0 \leq x \leq 1\ ,\ t > 0\ . \end{aligned}$$