ANALYSIS QUALIFYING EXAM

January 2002

Justify answers as completely as you can. Give careful statements of theorems you are using. **Time limit – 3 HOURS.**

1. Let $\Omega = \mathbb{C} - \{Non - Negative\ real\ numbers\}$. Is there a non-trivial (non-constant) bounded holomorphic function on Ω? Justify your answer.

2. Does there exist an analytic function mapping the annulus

$$A = \{z \mid 1 \leq |z| \leq 4\}$$

onto the annulus

$$B = \{z \mid 1 \leq |z| \leq 2\}$$

and taking $C_1 \to C_1$, $C_4 \to C_2$, where C_r is the circle of radius r?

3. Prove that a convex function on $\mathbb{R}$ is continuous.

4. If f is an entire holomorphic function, is there necessarily an entire holomorphic function g such that $e^g = f$? Prove your answer.

5. For real-valued functions on $\mathbb{R}$: If $f \in L1(\mathbb{R})$, (for the Lebesgue integral), find a necessary and sufficient condition that

$$\lim_{t \to 0} \frac{||f + tg||_1 - ||f||_1}{t}$$

exists for all $g \in L1(\mathbb{R})$.