Math 222 HW#2, due Thursday 2/11/21 **NAME:**

1. Problem 7-3: Jones (see book for hint/outline)
   The Lagrange identity
   $$||x||^2||y||^2 = (x \cdot y)^2 + ||x \times y||^2$$
   can be generalized. Using the outline Jones provides, prove that for $x, y, u, v \in \mathbb{R}^3$,
   $$(x \times y) \cdot (u \times v) = (x \cdot u)(y \cdot v) - (x \cdot v)(y \cdot u).$$

2. Problem 7-7: Jones
   In the definitions on page 9 under Chapter 7, D. Orientation, we have used the column vector representation of vectors. We may then write the corresponding $n \times n$ matrices
   $$\Phi = (\varphi_1 \ldots \varphi_n), \Psi = (\psi_1 \ldots \psi_n).$$
   Prove that the frames have the same orientation if and only if $\det \Phi$ and $\det \Psi$ have the same sign. Please do each direction of the iff separately for the sake of the grader.

3. Problem 7-9: Jones
   Prove that if $\{\varphi_1, \ldots, \varphi_n\}$ and $\{\psi_1, \ldots, \psi_n\}$ happen to be **orthonormal** frames, the matrix A in the second definition on page 9 under Chapter 7, D. Orientation is in $\mathrm{O}(n)$. Additionally prove that the frames have the same orientation if and only if $A \in \mathrm{SO}(n)$

4. Problem 7-13: Jones (in class problem)
   Using the formula on page 12, e.g.
   $$R(\hat{w}, \theta) = (\hat{u}\ \hat{v}\ \hat{w}) \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} (\hat{u}\ \hat{v}\ \hat{w})^{-1}$$
   show directly that
   $$R(\hat{w}, 0) = I.$$
   Then give a heuristic explanation of this equation.

5. Prof Jo level trig (in class problem)
   Let
   $$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$
   Find $\hat{w}$ and θ such that $= R(\hat{w}, \theta)$.

* Assignment Reflections
   How difficult was this assignment? How many hours did you spend on it? Which problems did you find to provide a worthwhile learning experience? Should I be assigning a similar number of problems, fewer problems, or more problems in the future? Is there a good mix of theory and computations?