Math 222 HW#3, due Friday 2/26/21 NAME:

1. Problem 8-10: Jones
   In the special case of an n-dimensional parallelogram P with edges $x_1, ..., x_n \in \mathbb{R}^n$, show that

$$\text{vol}_n(P) = |\det(x_1\ x_2\ \ldots\ x_n)|.$$

   (There is a substantial hint/outline available on Canvas under Discussions)

2. Folland
   Evaluate the following double integrals
   (a) $\int\int_S (x + 3y^3) dA,\ S =$ the upper half $y \geq 0$ of the unit disk $x^2 + y^2 \leq 1$
   (b) $\int\int_S (x^2 - \sqrt{y}) dA,\ S =$ the region between the parabola $y = x^2$ and the line $x = 2y$.

3. Folland
   Find the volume of the region above the triangle in the xy-plane with vertices $(0, 0)$, $(1, 0)$, and $(0, 1)$, and below the surface $z = 6xy(1 - x - y)$.

4. Folland
   For each of the following regions $S \subset \mathbb{R}^2$, express the double integral $\int\int_S f\ dA$ in terms of iterated integrals in two different ways, e.g. find the limits of integration for $dA = dxdy$ and $dA = dydx$. YOU DO NOT NEED TO EVALUATE THE INTEGRALS!
   (a) $S =$ the region in the left half plane between the curve $y = x^3$ and the line $y = 4x$.
   (b) $S =$ the triangle with vertices $(0, 0)$, $(2, 2)$, and $(3, 1)$.
   (c) $S =$ the region between the parabolas $y = x^2$ and $y = 6 - 4x - x^2$.

5. Folland
   Evaluate the following iterated integrals. (You may need to reverse the order of integration, in which case you must draw a labelled graph of the domain of integration).
   (a) $\int_1^3 \int_1^y ye^{2x}\ dx\ dy$
   (b) $\int_0^1 \int_{\sqrt{x}}^1 \cos(y^3 + 1)\ dy\ dx$
   (c) $\int_1^2 \int_{\frac{1}{x}}^1 ye^{xy}\ dy\ dx$

* Assignment Reflections
   How difficult was this assignment? How many hours did you spend on it? Which problems did you find to provide a worthwhile learning experience? Should I be assigning a similar number of problems, fewer problems, or more problems in the future? Is there a good mix of theory and computations?