Math 222 HW#4, due Sunday 3/7/21 **NAME:**

If you do the Folland Zero Content Proof (#1) you need not do #2 & #5 and you will still receive full credit. Alternatively, if you do #2-6, you need not do the Folland Zero Content proof (#1) and you will still receive full credit.

1. Folland Proof of Theorem 4.13 (Technical refinement of Theorem 4.12)
   Prove that if f is bounded on $[a, b]$ and the set of points in $[a, b]$ at which f is discontinuous has zero content, then f is integrable on $[a, b]$.

2. Stewart
   Use polar coordinates to combine the sum

$$\int_{\frac{1}{\sqrt{2}}}^{1} \int_{\sqrt{1-x^2}}^{x} xy \, dydx + \int_{1}^{\sqrt{2}} \int_{0}^{x} xy \, dydx + \int_{\sqrt{2}}^{2} \int_{0}^{\sqrt{4-x^2}} xy \, dydx$$

   into one double integral. Then evaluate the double integral.

3. Stewart & Jones 9.G

   (a) We define the improper integral (over the entire plane $\mathbb{R}^2$)

$$I = \iint_{\mathbb{R}^2} e^{-(x^2+y^2)} \, dA = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} \, dydx = \lim_{a \to \infty} \iint_{D_a} e^{-(x^2+y^2)} \, dA$$

   where D_a is the disk with radius a and center the origin. Show that

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} \, dA = \pi$$

   (b) An equivalent definition of the improper integral in part (a) is

$$\iint_{\mathbb{R}^2} e^{-(x^2+y^2)} \, dA = \lim_{a \to \infty} \iint_{S_a} e^{-(x^2+y^2)} \, dA$$

   where S_a is the square with vertices $(\pm a, \pm a)$. Use this to show that

$$\int_{-\infty}^{\infty} e^{-x^2} dx \int_{-\infty}^{\infty} e^{-y^2} dy = \pi$$

   (c) Deduce that

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

   (d) By making the change of variable $t = x\sqrt{2}$, show that

$$\int_{-\infty}^{\infty} e^{-x^2/2} dx = \sqrt{2\pi}$$

   (This is a fundamental result for probability and statistics).

4. Stewart & Jones 9.G Use the result of the previous exercise, part (c) to evaluate:

   (a) $\displaystyle\int_0^\infty x^2 e^{-x^2}\,dx$

   (b) $\displaystyle\int_0^\infty \sqrt{x}e^{-x}\,dx$

5. Stewart & Jones
   Use a triple integral to find the volume of the given solid. Also provide a sketch of the solid.

   (a) The tetrahedron enclosed by the coordinate planes and the plane $2x + y + z = 4$

   (b) The solid enclosed by the paraboloids $y = x^2 + z^2$ and $y = 8 - x^2 - z^2$.

6. Stewart & Jones
   Sketch the solid whose volume is given by the iterated integral. DO NOT EVALUATE!

   (a) $\displaystyle\int_0^1 \int_0^{1-x} \int_0^{2-2z} dydzdx$

   (b) $\displaystyle\int_0^2 \int_0^{2-y} \int_0^{4-y^2} dxdzdy$

* Assignment Reflections
How difficult was this assignment? How many hours did you spend on it? Which problems did you find to provide a worthwhile learning experience? Should I be assigning a similar number of problems, fewer problems, or more problems in the future? Is there a good mix of theory and computations?