Math 222 in class problems **Week: April 6, 2021** **Name:**

1. Find a parametrization for each of the following surfaces (hint: use an angular variable).
   (a) The surface obtained by revolving the curve $z = f(x)$, $a < x < b$ in the xz-plane around the z-axis, $a > 0$.
   (b) The surface obtained by revolving $z = f(x)$, $a < x < b$ in the xz-plane around the x-axis, $f(x) > 0$.
   (c) The lower sheet of the hyperboloid $z^2 - 2x^2 - y^2 = 1$.
   (d) The cylinder $x^2 + z^2 = 9$.

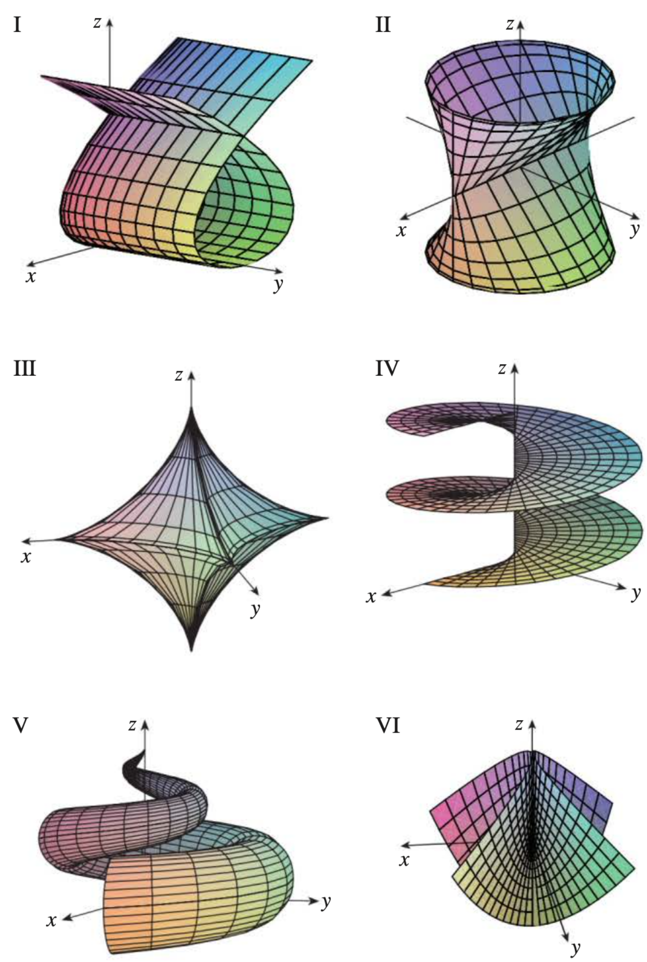
2. For each of the following maps $f : \mathbb{R}^2 \to \mathbb{R}^3$, describe the (possibly singular) surface $S = f(\mathbb{R}^2)$ and find a description of S as the locus of an equation $F(x, y, z) = 0$. Find the points where $\partial_u f$ and $\partial_v f$ are linearly dependent, and describe the singularities of S (if any) at these points.
   (a) $f(u, v) = (2u + v, u - v, 3v)$
   (b) $f(u, v) = (au \cos v, bu \sin v, u)$ with $a, b > 0$
   (c) $f(u, v) = (u \cos v, u \sin v, u^2)$

3. Find the a surface parametrization of the cap cut from the sphere $x^z + y^2 + z^2 = 4$ by the cone $z = \sqrt{x^2 + y^2}$ in terms of two variables. Give bounds on the two variables. Compute the surface area of said cap.

4. Find the surface area of the part of the paraboloid $z = x^2 + y^2$ inside the cylinder $x^2 + y^2 = a^2$

5. On HW # 9: Find the surface area of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. You should include a computation of why the surface area of a sphere of radius R is $4\pi R^2$.

6. Match the parametric equations with the surfaces.



13–18 Match the equations with the graphs labeled I–VI and give reasons for your answers. Determine which families of grid curves have u constant and which have v constant.

13. $\mathbf{r}(u, v) = u \cos v\,\mathbf{i} + u \sin v\,\mathbf{j} + v\,\mathbf{k}$

14. $\mathbf{r}(u, v) = uv^2\,\mathbf{i} + u^2v\,\mathbf{j} + (u^2 - v^2)\,\mathbf{k}$

15. $\mathbf{r}(u, v) = (u^3 - u)\,\mathbf{i} + v^2\,\mathbf{j} + u^2\,\mathbf{k}$

16. $x = (1 - u)(3 + \cos v) \cos 4\pi u$,
$y = (1 - u)(3 + \cos v) \sin 4\pi u$,
$z = 3u + (1 - u) \sin v$

17. $x = \cos^3 u \cos^3 v$, $y = \sin^3 u \cos^3 v$, $z = \sin^3 v$

18. $x = \sin u$, $y = \cos u \sin v$, $z = \sin v$