Math 222 in class problems Week: April 12, 2021 Name:

1. (Similar to HW #9) Use Stokes' theorem to evaluate $\int_C F \cdot \mathbf{dr}$ where $F(x, y, z) = \langle -y^2, x, z^2 \rangle$ and C is the curve of intersection of the plane $y + z = 2$ and the cylinder $x^2 + y^2 = 1$. Orient C to be counterclockwise when viewed from above. (Answer: π)

2. Use Stokes' theorem to compute $\iint_S \text{curl } F \cdot n \, dS$ where $F(x, y, z) = \langle xz, yz, xy \rangle$ and S is the portion of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy-plane. (The answer is 0.)

3. Jones Problem 13-6
   Let m be a fixed real number and let γ be the curve of intersection of the paraboloid $z = x^2 + y^2$ and the plane $z = mx$. Assume the curve has counterclockwise orientation as viewed from above, e.g. viewed from $(0, 0, r)$ for large positive r. Compute directly the line integral $\int_\gamma y \, dz$. Also compute the same line integral using Stokes' theorem. (Answer: $-\pi m^3/4$.)

4. Let C_r denote the circle of radius r about the origin in the xz-plane, oriented counterclockwise as viewed from the positive y-axis. Suppose F is a C^1 vector field on the complement of the y-axis in $\mathbb{R}^3$ such that $\int_{C_1} F \cdot \mathbf{dx} = 5$ and curl $F(x, yz) = 3\mathbf{j} + \dfrac{z\mathbf{i} - x\mathbf{k}}{(x^2 + z^2)^2}$. Compute $\int_{C_r} F \cdot \mathbf{dx}$ for every r. (Answer: $5 + 3\pi(r^2 - 1)$.)

5. Is there a vector field F on $\mathbb{R}^3$ such that curl $F = \langle x \sin y, \cos y, z - xy \rangle$?

6. Show that
$$\text{div } (F \times G) = G \cdot \text{curl } F - F \cdot \text{curl } G.$$