Math 222 in class problems **Week: February 8, 2021**

1. Problem 8-1: Jones
   Consider the "parallelogram" in $\mathbb{R}^3$ with "edges" equal to the three points $\mathbf{i}, \mathbf{j}, \mathbf{i} - 2\mathbf{j}$. Draw a sketch of it and conclude that it is actually a six-sided figure in the xy-plane.

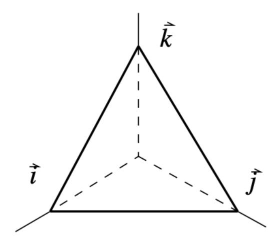
2. Problem 8-2: Jones
   Compute the area of the triangle in $\mathbb{R}^3$ whose vertices are $a\mathbf{i},\ b\mathbf{j},\ c\mathbf{k}$.
   Optional: Express the result in a form that is a symmetric function of a, b, c.

Hint for #2:

EXAMPLE. We find the area of the triangle in $\mathbb{R}^3$ with vertices $\vec{\imath}$, $\vec{\jmath}$, $\vec{k}$. We handle this by finding the area of an associated parallelogram and then dividing by 2 (really, 2!). A complication appears because we do not have the origin of $\mathbb{R}^3$ as a vertex, but we get around this by thinking of vectors emanating from one of the vertices (say $\vec{\jmath}$) thought of as the origin. Thus we take

$$x_1 = \vec{\imath} - \vec{\jmath} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \quad x_2 = \vec{k} - \vec{\jmath} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}.$$

Then the square of the area of the parallelogram is



$$\begin{aligned} \det(x_i \bullet x_j) &= \det \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \\ &= 3. \end{aligned}$$

Thus,

$$\text{area of triangle} = \frac{1}{2}\sqrt{3}.$$

Figure 1: Example on Jones 8-4