Choose your own Differential Adventure!

You are expected to TYPE up solutions to 3 problems with a connective theme which were not previously assigned for homework. These problems need not come from Guillemin-Pollack. You will then need to TYPE 4 paragraphs explaining your theme, highlighting the applicable concepts and theorems and how this connects with your choice of problems. Each problem needs to have 1-2 complete sentences to explain how it fits in. Your paragraphs can be interspersed throughout your write up and include background material.

Dates:
Proposal: due Friday April 4/10/26 at 11pm (late submission thru 4/12/26)
Complete write up: due Friday 5/1/26 at noon (early submission encouraged, late submission not possible)
Location: Gradescope

Comments on your proposals from Prof Jo will be available by Monday 4/13/26 at 11pm through gradescope. Late submissions will result in delayed feedback from Prof Jo.

Your proposal must include fully typed out statements of each of your 3 selected problems and indicate from where they are in the book (or if they are external to the book, where you found them). Your proposal should include 3-4 sentences about your theme, namely, what is the theme, what are the names of the concepts you are using, and the statement of the main theorem(s) you are using. It is ok to paraphrase as long as the main concepts would be clear to another classmate. You may present the latter (concepts and theorems) as a list. You are welcome to come chat with me or Alejandro, Teddy, or Maxwell during office hours about your ideas for a theme and/or the problems you are thinking about picking out.

Grade Breakdown:

- The proposal counts as a 'free full points' HW '11' (You must turn it in by late deadline!) It is not droppable.
- The overall adventure counts as 25% of your final grade.
- The adventure will be graded out of 100 points:
  10 'free' points for a reasonable draft (e.g. .75-1 completed problems and 1-2 paragraphs)
  30 points on connecting theme
  60 points on solutions

The use of AI is not permitted for any portion of the proposal or adventure. I reserve the right to request that you present your differential adventure solutions to me without notes to establish that you understand it. If you cannot do so then you will not receive credit for the problem. Repeated issues will be considered honor code violations and you will receive no credit for the entire adventure.

You do not need to write up the proof of the main theorems, though you are encouraged to indicate ideas if they are used in the solutions of your problems. Definitions do not have to go all the way back to basics, e.g. you don't have to explain the notion of a manifold or differential form.

If you get stuck on a problem you can replace it with a different problem in the write-up. You must seek approval from me if you want to swap out more than one problem. You can also come talk to me or Teddy, Alejandro, or Maxwell during office hours for help and feedback on problems and the write up.

Collaboration:
You are **encouraged to work with other students** in the class on finding problems and themes. You may work on solutions to the problems together, but **you must write up the connecting themes separately and independently**. It is ok for you to have an "identical" problem list with other students. Please write the names of the students with whom you worked on the proposal or on the problems in the first sentence of your proposal/write up. As with the homework, you are expected to write up the solutions on your own and write up the theme on your own. If you use materials beyond the course book or notes, you should cite/reference them and indicate this in your proposal.

Sample themes:
Here are some ideas to get you started. Feel free to modify these as you see fit or come up with your own. Some themes are more involved than others - pick one that interests you. :)

AI or data science
Differential geometry is useful to Machine Learning and data science. You can explain how it is useful and do some calculations in examples. A few of you have papers in mind relating to research you are doing. If you want to explore Riemannian geometry, the first edition of Lee's Riemannian Manifolds is a great resource, but the second edition will not be:
https://webhomes.maths.ed.ac.uk/~v1ranick/papers/leeriemm.pdf

Applications to Physics and General Relativity
The book, *Curvature of Space and Time, with an Introduction to Geometric Analysis* by Iva Stavrov is a great starting resource. See Prof Jo if you want a PDF. If you want to use physics textbooks that is cool too.

Totally Stoke'd and the Divergence Theorem
Exercises: 4.4.13, 4.4.14, 4.7.3, 4.7.4, 4.7.5, 4.7.6m
Theme: Explore Stokes theorem in many contexts, including the Divergence theorem in physics.

De Rham cohomology
Exercises: 4.6.1, 4.6.2, Problems from Lee.
Theme: Understand the relation between topology and manifolds. Lee has some great problems and more detailed exposition. This can be done at varying levels of mathematical complexity, depending on if you have taken a course in homology or not.

Connections between integration and mappings
Exercises: 4.8.1, 4.8.2, 4.8.3 (any 3 subproblems count as a 'whole' adventure problem), 4.8.5 (need DeRham cohomology), 4.8.8 (need cohomology)
Theme: Relate integration (analysis) to topological behavior of mappings (degree) as in 4.8. Explore the relationship with complex analysis by way of exercise 4.8.3 or cohomology by way of 4.8.8.

Measure 0 and Sard's Theorem
Exercises: 1.7.1 (exercise bottom of pg 202), 1.7.7 (Mini-Sard on pg 205), 1.7.2, 1.7.4, 1.7.6
Theme: Explore the connection with measure theory and understand (portions of the) proof of Sard's theorem in Appendix 1 and §1.7.

One Manifolds and some consequences
Exercises 2.1.1, 2.1.2, 2.1.3, 2.1.4, 2.1.7
Theme: Understand and prove some neat facts using manifolds in §2.2 and understand the classification of compact 1-manifolds in Appendix 2.