Math 451-551 HW #2: Due in Gradescope on Friday 1/23/26 NAME:

Please start each question on a separate page from the previous question.

1. Lee Proposition 2.10, Exercise 2.11 [Second Edition]
   Let M, N, P be smooth manifolds without boundary. Prove the following:

   (a) Every constant map $c : M \to N$ is smooth.

   (b) The identity map of M is smooth.

   (c) If $U \subset M$ is an open submanifold without boundary, then the inclusion map $i : U \hookrightarrow M$ is smooth. *(See Exercise 1.44 for the definition of open submanifold without boundary.)*

   To see a flavor of how these arguments go, read the proof of (d) in Lee.

2. Lee 3-1 [Second Edition]
   Suppose M and N are smooth manifolds without boundary and $F : M \to N$ is a smooth map. Show that $dF_p : T_pM \to T_{F(p)}N$ is the zero map for each $p \in M$ if and only if F is constant on each component.

3. Lee 3-6 [Second Edition]
   Consider $\mathbb{S}^3$ as the unit sphere in $\mathbb{C}^2$ under the usual identification $\mathbb{C}^2 \leftrightarrow \mathbb{R}^4$. For each $z = (z^1, z^2) \in \mathbb{S}^3$, define a curve $\gamma_z : \mathbb{R} \to \mathbb{S}^3$ by $\gamma_z(t) = (e^{it}z^1, e^{it}z^2)$. Show that γ_z is a smooth curve whose velocity is never zero.

4. Please spend some time reading Chapters 2-2 of Lee. Write down something interesting you learned or an example you liked. In Chapter 2, you can omit as much of the reading about partitions of unity as you would like. We won't use partitions of unity until we talk about integration and Riemannian metrics (at least 1 month away). In Chapter 2, the book alludes to the difficulties in distinguishing between smooth structures up to diffeomorphism. This is a subtle point I hope to explain more about as the semester progresses.

* Which problems provided a worthwhile learning experience? How many hours did you spend on it?