Choose your own Differential Adventure!

You are expected to TYPE up solutions to 3 problems with a connective theme which were not previously assigned for homework. These problems need not come from Montiel-Ros. Up to 2 of 3 of these problems may have book hints. (If you do select problems with hints you will need to justify every step.) You will then need to TYPE 2-4 paragraphs explaining your theme, highlighting the applicable concepts and theorems and how this connects with your choice of problems. Each problem needs to have a complete sentence to explain how it fits in. Your paragraphs can be interspersed throughout your write up.

Dates:
Proposal: due Tuesday November 11/26 at 11pm (earlier submission encouraged)
Polished draft: due Wednesday 12/4 at 11pm
Complete write up: due Monday 12/9 at 11pm (Prof Jo is slightly flexible if absolutely necessary)
Location: Gradescope

Comments on your proposals will be available by Thursday 11/28 at 11pm through gradescope. Comments will only be given on the draft upon request during Edgar or Prof Jo's office hours.

Your proposal must include fully typed out statements of each of your 3 selected problems and indicate from where they are in the book and if hints are given (or if they are external to the book, where you found them). Your proposal should include 3-4 sentences about your theme, namely, what is the theme, what are the names of the concepts you are using, and the statement of the main theorem(s) you are using. It is ok to paraphrase as long as the main concepts would be clear to another classmate. You may present the latter (concepts and theorems) as a list. You are welcome to come chat with me or Edgar during office hours about your ideas for a theme and/or the problems you are thinking about picking out.

Grade Breakdown:

- The proposal counts as a 'free full points' HW 10 (you must turn it in though on time!)
- The overall adventure counts as 30% of your final grade.
- The adventure will be graded out of 100 points:
  - 10 'free' points for a reasonable draft (e.g. 1.5-2.5 completed problems and 2 paragraphs)
  - 30 points on connecting theme
  - 60 points on solutions

You do not need to write up the proof of the main theorems, though you are encouraged to indicate ideas if they are used in the solutions of your problems. Definitions do not have to go all the way back to basics, e.g. it would be ok to assume I know what the Gauss curvature is, but if you need to use k_1 and k_2 then you should probably tell me at some point that the Gauss curvature can be expressed in terms of their product.

If you get stuck on a problem you can replace it with a different problem in the write-up. You must seek approval from me if you want to swap out more than one problem. You can also come talk to me or Edgar during office hours for help and feedback on problems and the write up.

Collaboration:
You are encouraged to work with other students in the class on finding problems and themes. You may work on solutions to the problems together, but you must write up the connecting themes separately and independently. It is ok for you to have an "identical" problem list with other students. Please write the names of the students with whom you worked on the proposal or on the problems in the first sentence of your proposal/write up. As with the homework, you are expected to write up the solutions on your own and write up the theme on your own. If you use materials beyond the course book or notes, you should cite/reference them and indicate this in your proposal.

Sample themes:
Here are some ideas to get you started. Feel free to modify these as you see fit.

Ode to a pringle
Discuss characterizations of the pringle (requires going beyond textbook). Explain the basics of hyperbolic 2-space, $\mathbb{H}^2$. Pick some exercises from https://e.math.cornell.edu/people/mann/papers/DIYhyp.pdf or the textbook: 3 (19), explicitly compute K and H of the pringle $z = x^2 - y^2$ using §3.6 (would count as 2 problems).

Ode to a sphere
Discuss characterizations of a sphere. Explain the rigidity result for spheres from §7.3 and other applications of the Hilbert-Liebmann theorem in §3.6, 7(12), 3.50, 3.51, 3 (13), 3 (19)

Euclidean vs. non-euclidean geometry
Explain how 0 Gauss curvature gives rise to euclidean geometry, Gauss curvature -1 gives rise to hyperbolic geometry and Gauss curvature +1 gives rise to spherical geometry. Explain the features of these geometries. (start with wikipedia). Pick some exercises from https://e.math.cornell.edu/people/mann/papers/DIYhyp.pdf

Non-positive Gauss curvature
Exercises 3.32, 3.33, 3.40, 3.42, 3 (2), 3 (7), 3 (19)
Theme: Explain how zero Gauss curvature forces the existence of a flat direction. Investigate why non-positive Gauss curvature implies the surface cannot be compact

Isometries of the flat plane
(*Look at Escher for inspiration.*) Explain the group structure for isometries of the plane (start with wikipedia). Contrast with isometry groups of surfaces and explain why the isometry group of a sphere consists of rigid motions; cf. §7.3.

Gauss Curvature
Exercises 3.40, 3.42, 3.51, 3 (19), 5.11, §7, etc
Theme: Explain the evolution of tools to study mean curvature over the semester and/or your favorite things about Gauss Curvature.

Mean Curvature
Exercises 3.50, 3 (2), 3(17), 3(18), 5(15), 6.14, 6.15, 6 (8), etc
Theme: Investigate Mean Curvature, try to read some of §6 if you feel adventurous.

Minimal surfaces

Exercises: 3.23 (Lagrange only got this far in his study of minimal surfaces), 3 (9), Show that a surface that locally minimizes its area is equivalent to it having zero mean curvature, explain how minimal surfaces can be defined in several equivalent ways. Show one of the triply periodic surfaces in 2 (8) is a minimal surface. Show that a catenoid or a helicoid is a minimal surface. Explain what the Enneper surface is and why it is a minimal surface. Explain how soap films show up.
Theme: Explain the implications of 0 mean curvature, and how this corresponds to surfaces which minimize area. What are some of the ways of characterizing minimal surfaces?
https://en.wikipedia.org/wiki/Minimal_surface
http://page.mi.fu-berlin.de/polthier/booklet/intro.html
https://math.byu.edu/~mdorff/docs/PaperBookMinSurfChapter.pdf
https://arxiv.org/pdf/2101.01375.pdf

Hyperbolic space

(*Look at Escher for inspiration.*) Explain the Poincare disk model (start with wikipedia). Explain the relation to euclidean (planar) geometry. Provide example(s) of hyperbolic surface. Investigate hyperbolic triangles. Pick some exercises from https://e.math.cornell.edu/people/mann/papers/DIYhyp.pdf

Geodesic triangles

Explain how the sum of the angles of a geodesic triangle varies with respect to the curvature of the surface Find some exercises to do about geodesics in §7. Explain the mathematics in the first part of §8.1. Pick some exercises from https://e.math.cornell.edu/people/mann/papers/DIYhyp.pdf

Visualizations of non-euclidean geometry

(*Look at Escher for inspiration.*) Explain the Poincare disk model. Draw some semi-regular or regular tilings of $\mathbb{R}^2$ and $\mathbb{H}^2$. (Each drawing counts as a problem; these can be hand drawn or computer assisted.) See Day 4 of https://e.math.cornell.edu/people/mann/papers/DIYhyp.pdf

Integrals

Exercises 6 (7), 6 (9), 6 (16), 6.14, 6.15, 6 (20), etc
Theme: Explain why the divergence theorem is useful.

Applications to physics with the divergence theorem

Exercises 5.35 (Archimedes' principle), 5 (11) Laplacian, 6 (18) Laplacian, 6 (19), etc
Theme: Tell me your favorite application(s) of the divergence theorem in physics, and why the Laplacian is important. Teach me some physics! You can use your favorite physics textbook(s).