GeoTop Final, due Monday 12/16/19 at 5pm **NAME:**

This is a 5 hour open notes exam. You may use any of the listed course textbooks but are not permitted to use the internet or material beyond the course textbooks. You are not permitted to speak with or email anyone about this exam except me. Write and sign the Rice honor pledge at the end of the exam.

Honor pledge: On my honor, I have neither given nor received any unauthorized aid on this exam.

1. Lee 5-7.
   Let $F : \mathbb{R}^2 \to \mathbb{R}$ be defined by $F(x, y) = x^3 + xy + y^3$. Which level sets of F are embedded submanifolds of $\mathbb{R}^2$? For each level set, prove either that it is or that it is not an embedded submanifold.

2. Lee 6-11.
   Suppose $F : M \to N$ and $G : N \to P$ are smooth maps, and G is transverse to an embedded submanifold $X \subset P$. Show that F is transverse to the submanifold $G^{-1}(X)$ if and only if $G \circ F$ is transverse to X.

3. Lee 16-2.
   Let $T^2 = S^1 \times S^1 \subset \mathbb{R}^4$ denote the 2-torus, defined as the set of points (w, x, y, z) such that $w^2 + x^2 = y^2 + z^2 = 1$, with the product orientation determined by the standard orientation on S^1 (e.g. don't worry about it). Compute $\int_{T^2} \omega$, where ω is the following 2-form on $\mathbb{R}^4$:
   $$\omega = xyz \, dw \wedge dy.$$

4. A symplectic manifold is a smooth manifold M equipped with a nondegenerate closed 2-form ω. A closed nondegenerate 2-form is said to be a symplectic form.
   (a) Show that if there exists a symplectic form on a smooth manifold M, then dim $M = 2n$.
   (b) Show that the only sphere S^n which admits a symplectic form is S^2.
   Hint: Use Stokes' theorem and the computation of the de Rham cohomology of S^n.

*math: Lee 16-9
Let ω be the $(n-1)$-form on $\mathbb{R}^n \setminus \{0\}$

$$\omega = |x|^{-n} \sum_{i=1}^{n} (-1)^{i-1} x^i \, dx^1 \wedge ... \wedge \widehat{dx^i} \wedge ... \wedge dx^n.$$

(a) Show that $\iota_{S^{n-1}}^* \omega$ is the Riemannian volume form of S^{n-1} with respect to the round metric and the standard orientation.
(b) Show that ω is closed but not exact on $\mathbb{R}^n \setminus \{0\}$.