Math 444/539 HW#11, due Friday 12/6/19 NAME:

1. Lee 14.6 SECOND
   Define a 2-form ω on $\mathbb{R}^3$ by

$$\omega = x dy \wedge dz + y dz \wedge dx + z dx \wedge dy.$$

   (a) Compute ω in spherical coordinates (ρ, φ, θ) defined by

$$(x, y, z) = (\rho \sin\varphi \cos\theta, \rho \sin\varphi \sin\theta, \rho \cos\varphi).$$

   (b) Compute $d\omega$ in both Cartesian and spherical coordinates and verify that both expressions represent the same 3-form.

   (c) Compute the pullback $\iota^*_{S^2}\omega$ to S^2, using coordinates (φ, θ) on the open subset where these coordinates are defined.

   (d) Show that $\iota^*_{S^2}\omega$ is nowhere zero.

2. Lee 14.7 SECOND
   Let $M = \mathbb{R}^2$ and $N = \mathbb{R}^3$, $\omega = y dz \wedge dx$, and $F : M \to N$ is the smooth map defined by

$$F(\theta, \varphi) = \left((\cos\varphi + 2)\cos\theta, (\cos\varphi + 2)\sin\theta, \sin\varphi\right).$$

   Compute $d\omega$ and $F^*\omega$, and verify by direct computation that $F^*(d\omega) = d(F^*\omega)$.

3. Define a 1-form α on the punctured plane $\mathbb{R}^2 \setminus \{0\}$ by

$$\alpha = \left(\frac{-y}{x^2 + y^2}\right) dx + \left(\frac{x}{x^2 + y^2}\right) dy.$$

   (a) Calculate $\int_C \alpha$ for any circle C of radius r around the origin.

   (b) Prove that in the half plane $\{x > 0\}$, α is the differential of a function.

4. Lee 16.10 SECOND
   Let D denote the torus of revolution in $\mathbb{R}^3$ obtained by revolving the circle $(r-2)^2 + z^2 = 1$ around the z-axis (example 5.17), with its induced Riemannian metric and with the orientation determined by the outward unit normal.

   (a) Compute the surface area of D

   (b) Compute the integral over D of the 2-form $\omega = z dx \wedge dy$.

everyone: How difficult was this assignment? How many hours did you spend on it?