Math 401 HW#1, due Wednesday 9/4/24 **NAME:**

Do not turn in the ⋆ in class problems. But make sure that you finished and understand them!

⋆ **Multi Warm Up (in class)**
Let $v(t) \in \mathbb{R}^n$ such that $|v(t)| = c$ for all t. Show $v' \cdot v = 0$. Conclude $T(t) \cdot N(t) = 0$.

⋆ **Multi Warm Up: Jones Problem 11-1 (in class)**
Suppose $C \subset \mathbb{R}^2$ is a curve described in polar coordinates by an equation $r = g(\theta)$, where $a \leq \theta \leq b$. Show that the length of C is

$$\int_a^b \sqrt{g'(\theta)^2 + g(\theta)^2}\, d\theta.$$

Remark: This formula is usually written as $\int_a^b \sqrt{\left(\frac{dr}{d\theta}^2 + r^2\right)}\, d\theta.$

1. **Exercise 1.12** (the multi-warm up will be advantageous)
   Consider the logarithmic spiral $\alpha : \mathbb{R} \to \mathbb{R}^2$ given by

   $$\alpha(t) = (ae^{bt}\cos t, ae^{bt}\sin t)$$

   with $a > 0$, $b < 0$. Compute the arc length function $S : \mathbb{R} \to \mathbb{R}$ where $S(t) = \int_{t_0}^t |\alpha'(\tau)| d\tau$, where t_0 corresponds to an arbitrary choice of $t_0 \in \mathbb{R}$. Reparametrize by arclength (your formula will not be pretty). Describe/sketch the trace of this curve.

2. **Remembering Linear Algebra**
   (a) Provide a counterexample to the claim: For every square matrix B, $||Bv|| = |\det B|\ ||v||$.
   (b) Given an orthogonal matrix $A \in \mathrm{O}(n)$, show that A preserves vector norms: $||Av|| = ||v||$.
   (Recall that an orthogonal matrix A satisfies $AA^T = A^T A = \mathrm{Id}$.)

3. **Exercise 1.8**
   Let $\alpha : I \to \mathbb{R}^3$ be a curve and let $M : \mathbb{R}^3 \to \mathbb{R}^3$ be a rigid motion, e.g. $M = Ax + b$, where $A \in \mathrm{O}(3)$ and $b \in \mathbb{R}^3$ is a fixed vector. Prove that rigid motions preserve the length of curves, namely $L_b^a(\alpha) = L_b^a(M \circ \alpha)$.

4. **Exercise 1.12**
   Let $\phi : J \to I$ be a diffeomorphism and let $\alpha : I \to \mathbb{R}^3$ be a curve. Given $[a, b] \subset J$ with $\phi([a, b]) = [c, d]$ prove that $L_a^b(\alpha \circ \phi) = L_c^d(\alpha)$.

* **Assignment Reflections**
How long did this take you? How was the difficulty? Which problems were meaningful?