1. Show that a space which is connected and locally path connected is path connected.
Recall that a subset $A$ of a topological space $X$ is called a retract of $X$ if there exists a continuous map $r : X \to A$ (called a retraction) such that $r(a) = a$ for any $a \in A$.

2. Prove that the relation “is a retract of” is transitive, i.e. if $A$ is a retract of $B$ and $B$ is a retract of $C$, then $A$ is a retract of $C$. 
Definition: A subspace $A \subset X$ is called a **strong deformation retract** of $X$ if there exists a homotopy $F : X \times I \to X$ such that

\[
\begin{align*}
F(x, 0) &= x \\
F(x, 1) &\in A \\
F(a, t) &= a \quad \text{for } a \in A \text{ and all } t \in I.
\end{align*}
\]

The subspace $A$ is merely a **deformation retract** if the last equation holds only when $t = 1$.

3. Show that a deformation retract of a Hausdorff space must be a closed subset.
4. Give an example of a space which is connected but not path connected. Be sure to show that it is in fact connected but not path connected.

*Hint:* *What can you do to the graph* \( y = \sin \left( \frac{1}{x} \right) \)? **DO NOT GOOGLE THIS.**
Prove that if \( A \) is a retract of a topological space \( X \), \( r : X \to A \) is a retraction, \( i : A \hookrightarrow X \) is inclusion, and \( i_*(\pi_1(A)) \) is a normal subgroup of \( \pi_1(X) \), then \( \pi_1(X) \) is the direct product of the subgroup image \( i_* \) and kernel \( r_* \).
everyone: How difficult was this assignment? How many hours did you spend on it? Indicate if you are math graduate student or not.