Math 444/539 HW#1, due Wednesday 9/4/19 NAME:

1. Show that a space which is connected and locally path connected is path connected.

Recall that a subset A of a topological space X is called a *retract* of X if there exists a continuous map $r : X \to A$ (called a retraction) such that $r(a) = a$ for any $a \in A$.

2. Prove that the relation "is a retract of" is transitive, i.e. if A is a retract of B and B is a retract of C, then A is a retract of C.

Defintion: A subspace $A \subset X$ is called a **strong deformation retract** of X if there exists a homotopy $F : X \times I \to X$ such that

$$\begin{aligned} F(x,0) &= x \\ F(x,1) &\in A \\ F(a,t) &= a \qquad \text{for } a \in A \text{ and all } t \in I. \end{aligned}$$

The subspace A is merely a **deformation retract** if the last equation holds only when $t = 1$.

3. Show that a deformation retract of a Hausdorff space must be a closed subset.

4. Give an example of a space which is connected but not path connected. Be sure to show that it is in fact connected but not path connected.
   Hint: What can you do to the graph $y = \sin\left(\frac{1}{x}\right)$*?* **DO NOT GOOGLE THIS.**

*math: Prove that if A is a retract of a topological space X, $r : X \to A$ is a retraction, $i : A \hookrightarrow X$ is inclusion, and $i_*(\pi_1(A))$ is a normal subgroup of $\pi_1(X)$, then $\pi_1(X)$ is the direct product of the subgroup image i_* and kernel r_*.

everyone: How difficult was this assignment? How many hours did you spend on it? Indicate if you are math graduate student or not.