Math 444/539 HW#3, due Monday 9/16/19 **NAME:**

1. Find $\pi_1(X)$ of each of the following spaces. The problems are independent of each other.
   (a) Let X be the union of S^2 with the unit disk in the xy-plane.
   (b) Let X be the union of S^2 with the straight line segment connecting the north pole to the south pole.

2. Show that there are no retractions $r : X \to A$ in the following cases

   (a) $X = \mathbb{R}^3$ with A any subspace homeomorphic to S^1.

   (b) $X = S^1 \times D^2$ with A its boundary torus $S^1 \times S^1$.

   (c) $X = D^2 \vee D^2$ with A its boundary $S^1 \vee S^1$.
   (See page 10 of Hatcher for the definition of the wedge product '$\vee$'.)

3. If K^2 is the Klein bottle (https://en.wikipedia.org/wiki/Klein_bottle), compute $\pi_1(K^2)$, using Seifert van Kampen. (It's ok to also look in Massey).

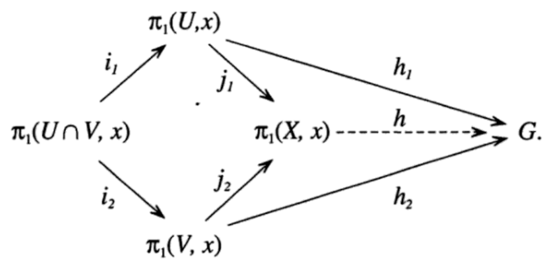
4. Let F denote the free group on two generators a and b, and let $G = [F, F]$ be the commutator subgroup of F. Show that G is freely generated by

$$\{[a^m, b^n] \mid m, n \in \mathbb{Z} \setminus \{0\}\}$$

5. Prove the following Proposition.

Proposition 5.3 (Massey Chapter III)
Let F be a free group on the set S with respect to a function $\varphi : S \to F$ and let $\pi : F \to F/[F,F]$ denote the natural projection of F onto the quotient group. Then, $F/[F,F]$ is a free abelian group on S with respect to the function $\pi\varphi : S \to F/[F,F]$.

*math: If V is simply connected show that the inclusion map $j_1 : \pi_1(U, x) \to \pi_1(X, x)$ is surjective, with kernel the smallest normal subgroup of $\pi_1(X, x)$ that contains the image of the inclusion map $i_1 : \pi_1(U \cap V, x) \to \pi_1(U, x)$.



everyone: How difficult was this assignment? How many hours did you spend on it?